The Math Resource for Instruction for

North Carolina Math 3

NC Mathematics
Department of Public Instruction

PUBLIC SCHOOLS OF NORTH CAROLINA
State Board of Education
Department of Public Instruction

Link for: Feedback for NC's Math Resource for Instruction

Link to: Suggest Resources for NC's Math Resource for Instruction

North Carolina Math 3 Standards

Number

The complex number system
Use complex numbers in polynomial identities and equations
NC.M3.N-CN.9

Algebra

Overview

Seeing structure in expressions
Interpret the structure of expressions
NC.M3.A-SSE.1a
NC.M3.A-SSE.1b
NC.M3.A-SSE.2
Write expressions in equivalent form to solve problems
NC.M3.A-SSE.3c

Perform arithmetic operations on polynomials
Understand the relationship between zeros and the factors of polynomials
NC.M3.A-APR.2
NC.M3.A-APR.3
Rewrite rational expressions
NC.M3.A-APR.6
NC.M3.A-APR.7a
NC.M3.A-APR.7b

Creating equations
Create equations that describe numbers or relationships
NC.M3.A-CED.1
NC.M3.A-CED.2
NC.M3.A-CED.3

Reasoning with equations and inequalities
Understand solving equations as a process of reasoning and explain the reasoning
NC.M3.A-REI.1
NC.M3.A-REI.2

Represent and solve equations and inequalities graphically
NC.M3.A-REI.11

Functions

Overview

Interpreting functions
Understand the concept of a function and use function notation
NC.M3.F-IF.1
NC.M3.F-IF.2
Interpret functions that arise in applications in terms of a context
NC.M3.F-IF.4
Analyze functions using different representations
NC.M3.F-IF.7
NC.M3.F-IF.9

Building functions
Build a function that models a relationship between two quantities
NC.M3.F-BF.1a
NC.M3.F-BF.1b
Build new functions from existing functions
NC.M3.F-BF.3
NC.M3.F-BF.4a
NC.M3.F-BF.4b
NC.M3.F-BF.4c

Linear, Quadratic and Exponential Models
Construct and compare linear and exponential models to solve problems
NC.M3.F-LE.3
NC.M3.F-LE.4

Trigonometric Functions
Extend the domain of trigonometric functions using the unit circle
NC.M3.F-TF.1
NC.M3.F-TF.2a
NC.M3.F-TF.2b
Model periodic phenomena with trigonometric functions
NC.M3.F-TF.5

Geometry

Overview

Congruence
Prove geometric theorems
NC.M3.G-CO.10
NC.M3.G-CO.11
NC.M3.G-CO.14

Circles
Understand and apply theorems about circles
NC.M3.G-C.2
NC.M3.G-C.5

Expressing Geometric Properties with Equations
Translate between the geometric description and the equation for a conic section
NC.M3.G-GPE.1

Geometric Measurement & Dimension
Explain volume formulas and use them to solve problems
NC.M3.G-GMD.3
Visualize relationships between two-dimensional and three-dimensional objects
NC.M3.G-GMD.4

Modeling with Geometry
Apply geometric concepts in modeling situations
NC.M3.G-MG.1

Statistics & Probability

Overview

Making Inference and Justifying Conclusions
Understand and evaluate random processes underlying statistical experiments
NC.M3.S-IC.1
Making inferences and justify conclusions from sample surveys, experiments and observational studies
NC.M3.S-IC.3
NC.M3.S-IC.4
NC.M3.S-IC.5
NC.M3.S-IC.6

NC.M3.N-CN.9

Use complex numbers in polynomial identities and equations.

Use the Fundamental Theorem of Algebra to determine the number and potential types of solutions for polynomial functions.

Concepts and Skills

Pre-requisite

- Understand the relationship between the factors and the zeros of a polynomial function (NC.M3.A-APR.3)

Connections

- Interpret parts of an expression (NC.M3.A-SSE.1a)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Multiply and divide rational expressions (NC.M3.A-APR.7b)
- Creating equations to solve or graph (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Justify a solution method and the steps in the solving process (NC.M3.A-REI.1)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11)
- Finding and comparing key features of functions (NC.M3.F-IF.4, 7, 9)
- Building functions from graphs, descriptions and ordered pairs (NC.M3.F-BF.1a)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

3 – Construct viable arguments and critique the reasoning of others

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss how can you determine the number of real and imaginary solutions of a polynomial.

New Vocabulary: The Fundamental Theorem of Algebra

Mastering the Standard

Comprehending the Standard

Students know The Fundamental Theorem of Algebra, which states that *every polynomial function of positive degree n has exactly n complex zeros (counting multiplicities).* Thus a linear equation has 1 complex solution, a quadratic has two complex solutions, a cubic has three complex solutions, and so on. The zeroes do not have to be unique. For instance $(x-3)^2=0$ has zeroes at $x=3$ and $x=3$. This is considered to have a double root or a multiplicity of two.

Assessing for Understanding

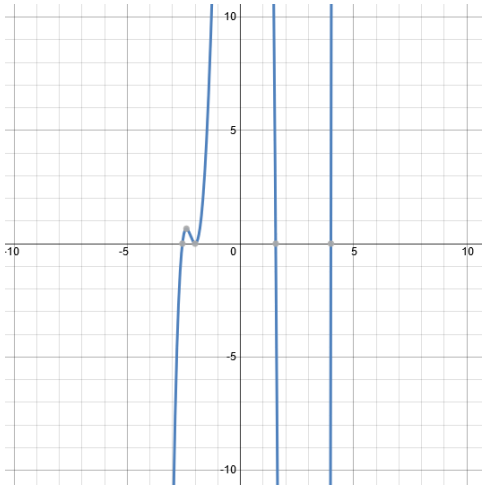
First, students need to be able to identify the number of solutions to a function by relating them to the degree.

Example: How many solutions exist for the function $(x) = x^4 - 10x + 3$?

Going deeper into the standard, students need to determine the types of solutions using graphical or algebraic methods, where appropriate.

Example (*real and imaginary solutions*): How many, and what type, of solutions exist for the function $(x) = x^4 - 10x^2 - 21x - 12$?

Example (*with multiplicity of 2*): How many, and what type, of solutions exist for the function $f(x) = x^5 - 3x^4 - 27x^3 + 19x^2 + 114x - 72$?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
Students also understand the graphical (x-intercepts as real solutions to functions) and algebraic (solutions equal to zero by methods such as factoring, quadratic formula, the remainder theorem, etc.) processes to determine when solutions to polynomials are real, rational, irrational, or imaginary.	**Example**: What is the lowest possible degree of the function graphed below? How do you know? What is another possible degree for the function? 

Instructional Resources	
Tasks	**Additional Resources**
	Algebra made Fundamental (CPalms) NEW Truncated Graph NEW Representing Polynomials Graphically NEW

Back to: Table of Contents

Algebra, Functions & Function Families

NC Math 1	NC Math 2	NC Math 3
Functions represented as graphs, tables or verbal descriptions in context		
Focus on comparing properties of linear function to *specific* non-linear functions and rate of change. • Linear • Exponential • Quadratic	**Focus on properties of quadratic functions and an introduction to inverse functions through the inverse relationship between quadratic and square root functions.** • Quadratic • Square Root • Inverse Variation	**A focus on more complex functions** • Exponential • Logarithm • Rational functions w/ linear denominator • Polynomial w/ degree $\leq$ three • Absolute Value and Piecewise • Intro to Trigonometric Functions
A Progression of Learning of Functions through Algebraic Reasoning		
The conceptual categories of Algebra and Functions are inter-related. Functions describe situations in which one quantity varies with another. The difference between the Function standards and the Algebra standards is that the Function standards focus more on the characteristics of functions (e.g. domain/range or max/min points), function definition, etc. whereas the Algebra standards provide the computational tools and understandings that students need to explore specific instances of functions. As students progress through high school, the coursework with specific families of functions and algebraic manipulation evolve. Rewriting algebraic expressions to create equivalent expressions relates to how the symbolic representation can be manipulated to reveal features of the graphical representation of a function. **Note:** The Numbers conceptual category also relates to the Algebra and Functions conceptual categories. As students become more fluent with their work within particular function families, they explore more of the number system. For example, as students continue the study of quadratic equations and functions in Math 2, they begin to explore the complex solutions. Additionally, algebraic manipulation within the real number system is an important skill to creating equivalent expressions from existing functions.		

Back to: Table of Contents

Algebra – Seeing Structure in Expressions

NC.M3.A-SSE.1a

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

a. Identify and interpret parts of a piecewise, absolute value, polynomial, exponential and rational expressions including terms, factors, coefficients, and exponents.

Concepts and Skills

Pre-requisite

- Identify and interpret parts of an expression in context (NC.M2.A-SSE.1a)

Connections

- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Interpret parts of an expression as a single entity (NC.M3.A-SSE.1b)
- Create and graph equations and systems of equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Interpret one variable rational equations (NC.M3.A-REI.2)
- Interpret statements written in piecewise function notation (NC.M3.F-IF.2)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Understand the effects on transformations on functions (NC.M3.F-BF.3)
- Interpret inverse functions in context (NC.M3.F-IF.4c)
- Interpret the sine function in context (NC.M3.F-TF.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: Absolute value, piecewise function, rational function

Mastering the Standard

Comprehending the Standard

Students need to be able to determine the meaning, algebraically and from a context, of the different parts of the expressions noted in the standard. At the basic level, this would refer to identifying the terms, factors, coefficients, and exponents in each expression.

Students must also be able to identify how these key features relate in context of word problems.

Assessing for Understanding

Students should be able to identify and **explain the meaning** of each part of these expressions.

- **Example:** The Charlotte Shipping Company is needing to create an advertisement flyer for its new pricing for medium boxes shipped within Mecklenburg County. Based on the expressions of the function below, where *c* represents cost and *p* represent pounds, create an advertisement that discusses all the important details for the public. $\boldsymbol{c(p)} =$

$$\begin{cases} \mathbf{11.45}, & \boldsymbol{p \leq 12\frac{1}{3}} \\ \mathbf{.72\,p + 5.57}, & \boldsymbol{p > 12\frac{1}{3}} \end{cases}$$

- **Example:** In a newspaper poll, 52% of respondents say they will vote for a certain presidential candidate. The range of the actual percentage can be expressed by the expression $|x - 4|$, where *x* is the actual percentage. What are the highest and lowest percentages that might support the candidate? Is the candidate guaranteed a victory? Why or why not?

Instructional Resources	
Tasks	**Additional Resources**
	Rational Functions Unit Classroom Task: 4.2 (Mathematics Visions Project) NEW

Back to: Table of Contents

Algebra – Seeing Structure in Expressions

NC.M3.A-SSE.1b

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

Concepts and Skills

Pre-requisite

- Interpret parts of a function as a single entity (NC.M2.A-SSE.1b)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a)

Connections

- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Create and graph equations and systems of equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Interpret one variable rational equations (NC.M3.A-REI.2)
- Interpret statements written in function notation (NC.M3.F-IF.2)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Understand the effects on transformations on functions (NC.M3.F-BF.3)
- Interpret inverse functions in context (NC.M3.F-IF.4c)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: piecewise function

Mastering the Standard

Comprehending the Standard

Students must be able to take the multi-part expressions we engage with in Math 3 and see the different parts and what they mean to the expression in context. Students have worked with this standard in Math 1 and Math 2, so the new step is applying it to our Math 3 functions.

As we add piecewise functions and expressions in Math 3, breaking down these expressions and functions into their parts are essential to ensure understanding.

For Example: Explain what operations are performed on the inputs -2, 0, and 2 for the following expression:

Assessing for Understanding

Students must be able to demonstrate that they can understand, analyze, and interpret the information that an expression gives in context. The two most important parts are determining what a certain situation asks for, and then how the information can be determined from the expression.

Example: The expression, $.0013x^3 - .0845x^2 + 1.6083x + 12.5$, represents the gas consumption by the United States in billions of gallons, where *x* is the years since 1960. Based on the expression, how many gallons of gas were consumed in 1960? How do you know?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
$f(x) = \begin{cases} 3x, for\ x < 0 \\ \frac{1}{x}, for\ 0 \leq x < 2 \\ x^3, for\ x \geq 2 \end{cases}$ Which input is not in the domain? Why not?	

Instructional Resources	
Tasks	**Additional Resources**
	Rational Functions Unit Classroom Task: 4.2 (Mathematics Visions Project) NEW

Back to: Table of Contents

Algebra – Seeing Structure in Expressions

NC.M3.A-SSE.2

Interpret the structure of expressions.

Use the structure of an expression to identify ways to write equivalent expressions.

Concepts and Skills

Pre-requisite

- Justifying a solution method (NC.M2.A-REI.1)

Connections

- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Write an equivalent form of an exponential expression (NC.M3.A-SSE.3c)
- Create and graph equations and systems of equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Justify a solution method (NC.M3.A-REI.1)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Analyze and compare functions for key features (NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make use of structure

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: Sum or Difference of Cubes

Mastering the Standard

Comprehending the Standard

In Math 1 and 2, students factored quadratics. In Math 3, extend factoring to include strategies for rewriting more complicated expressions. Factoring a sum or difference of cubes, factoring a GCF out of a polynomial, and finding missing coefficients for expressions based on the factors can all be included.

For Example: When factoring a difference of cubes, is the trinomial factor always, sometimes or never factorable? How do you know?

Assessing for Understanding

This standard can be assessed mainly by performing the algebraic manipulation. Problems could include:

Example: Factor $x^3 - 2x^2 - 35x$

Example: The expression $(x + 4)$ is a factor of $x^2 + kx - 20$. What is the value of k? How do you know?

Example: Factor $x^3 - 8$

Instructional Resources

Tasks	Additional Resources

Algebra – Seeing Structure in Expressions

NC.M3.A-SSE.3c

Write expressions in equivalent forms to solve problems.

Write an equivalent form of an exponential expression by using the properties of exponents to transform expressions to reveal rates based on different intervals of the domain.

Concepts and Skills

Pre-requisite

- Use the properties of exponents to rewrite expressions with rational exponents (NC.M2.N-RN.2)

Connections

- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Analyze and compare functions for key features (NC.M3.F-IF.7, NC.M3.F-IF.9)
- Building functions from graphs, descriptions and ordered pairs (NC.M3.F-BF.1a)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain their process of transforming an exponential expression using mathematical reasoning.

Mastering the Standard

Comprehending the Standard

Students have already learned about exponential expressions in Math 1. This standard expands on that knowledge to expect students to write equivalent expressions based on the properties of exponents.

Additionally, compound interest is included in this standard. In teaching students to fully mastery this concept, we must explain where the common compound interest formula originates. The relationship to the common $A = P(1 + r)^t$ formula must be derived and explained.

Assessing for Understanding

For students to demonstrate mastery, they must be able to convert these expressions and explain why the conversions work mathematically based on the properties of exponents.

Example: Explain why the following expressions are equivalent.

$2\left(\frac{1}{2}\right)^6 \qquad \left(\frac{1}{2}\right)^5 \qquad 2\left(\frac{1}{4}\right)^3$

Students must be able to convert an exponential expression to different intervals of the domain.

Example: In 1966, a Miami boy smuggled three Giant African Land Snails into the country. His grandmother eventually released them into the garden, and in seven years there were approximately 18,000 of them. The snails are very destructive and need to be eradicated.

a) Assuming the snail population grows exponentially, write an expression for the population, *p*, in terms of the number, *t*, of years since their release.

b) You must present to the local city council about eradicating the snails. To make a point, you want to want to show the rate of increase per month. Convert your expression from being in terms of years to being in terms of months.

Modified from Illustrative Mathematics
https://www.illustrativemathematics.org/content-standards/tasks/638

Instructional Resources	
Tasks	**Additional Resources**
	Compound Interest Introduction

Back to: Table of Contents

Algebra – Arithmetic with Polynomial Expressions

NC.M3.A-APR.2

Understand the relationship between zeros and factors of polynomials.

Understand and apply the Remainder Theorem.

Concepts and Skills

Pre-requisite

- Evaluate functions (NC.M1.F-IF.2)
- Division of polynomials (NC.M3.A-APR.6)

Connections

- Understand the relationship between the factors of a polynomial, solutions and zeros (NC.M3.A-APR.3)
- Create and graph equations (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Justify a solution method and the steps in the solving process (NC.M3.A-REI.1)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Building functions from graphs, descriptions and ordered pairs (NC.M3.F-BF.1a)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make use of structure
8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.
Students should be able to accurately explain Remainder Theorem in their own words.

Recalled Vocabulary: Divisor, Dividend, Quotient, Remainder

Mastering the Standard

Comprehending the Standard

Students must understand that the Remainder Theorem states that if a polynomial $p(x)$ is divided by any binomial $(x - c)$, *which does not have to be a factor of the polynomial*, the remainder is the same as if you evaluate the polynomial for c (meaning to evaluate $p(c)$). If the remainder $p(c) = 0$ then $(x - c)$ is a factor of $p(x)$ and c is a solution of the polynomial.
Students should be able to know and apply all of the Remainder Theorem. Teachers should not limit the focus to just finding roots.

Students can discover this relationship by completing the division and evaluating the function for the same value to see how the remainder and the function's value are the same.

Assessing for Understanding

Students should be able to apply the Remainder Theorem.

Example: Let $p(x) = x^5 - x^4 + 8x^2 - 9x + 30$. Evaluate $p(-2)$. What does the solution tell you about the factors of $p(x)$?

Solution: $p(-2) = 32$. This means that the remainder of $\frac{x^5 - x^4 + 8x^2 - 9x + 30}{x+2}$ is $\frac{32}{x+2}$. This also means that $x + 2$ is not a factor of $x^5 - x^4 + 8x^2 - 9x + 30$.

Example: Consider the polynomial function: $P(x) = x^4 - 3x^3 + ax^2 - 6x + 14$, where a is an unknown real number. If $(x - 2)$ is a factor of this polynomial, what is the value of a?

Instructional Resources

Tasks	Additional Resources
	Remainder Theorem Discovery

Back to: Table of Contents

Algebra – Arithmetic with Polynomial Expressions

NC.M3.A-APR.3

Understand the relationship between zeros and factors of polynomials.

Understand the relationship among factors of a polynomial expression, the solutions of a polynomial equation and the zeros of a polynomial function.

Concepts and Skills

Pre-requisite

- Understand the relationship between the linear factor of a quadratic expression and solutions and zeros (NC.M1.A-APR.3)

Connections

- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Create and graph equations (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Justify a solution method (NC.M3.A-REI.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Recalled Vocabulary: Divisor, Dividend, Quotient, Remainder

Mastering the Standard

Comprehending the Standard

In Math 1, students studied the relationships of factors, zeroes, and solutions as they related to quadratics. In Math 3, students will expand on these relationships to higher-order polynomials with more than two factors and/or solutions.

For Example: What relationship exists between factors of polynomials and their solutions? What type of solutions can exist when a polynomial is not factorable?

Note: It is not sufficient to allow students to use the shortcut that solutions are the "opposite" of the number in a binomial factor, because when the leading coefficient is greater than one, this is not true. (For example, the factor (2x + 3).) Students must understand that the relationship between the solutions of an equation and the zeros of a function and the multiplicative property of zero.

Assessing for Understanding

Students must understand how to set factors equal to 0 to solve a polynomial AND how to build factors from the solutions to a polynomial.

Example: What are the solutions to the polynomial: $p(x) = (x - 5)(3x + 5)(x^2 - 7x + 15)$

Example: Write two distinct polynomials, in factored form, with solutions at 1, $\frac{4}{3}$, and a double root at -4.

Instructional Resources

Tasks

Additional Resources

Representing Polynomials Graphically NEW

Back to: Table of Contents

Algebra – Arithmetic with Polynomial Expressions

NC.M3.A-APR.6

Rewrite rational expressions.

Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, where $a(x), b(x), q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$.

Concepts and Skills

Pre-requisite

- Long division of numerical expressions
  Operations with polynomial expressions (NC.M2.A-APR.1)

Connections

- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Operations with polynomial expressions (NC.M3.A-APR.7a, NC.M3.A-APR.7b)
- Create and graph equations (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Justify a solution method (NC.M3.A-REI.1)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

If students learn synthetic division, students should be able to describe the limitations of the process.

Recalled Vocabulary: Divisor, Dividend, Quotient, Remainder

Mastering the Standard

Comprehending the Standard

In teaching this standard, students must be able to divide and simplify rational expressions by factoring and simplifying (inspection) and long division. It will be important for students to realize when each can and should be used.

Note: The use of synthetic division may be introduced as a method but students should recognize its limitations (division by a linear term). When students use methods that have not been developed conceptually, they often create misconceptions and make procedural mistakes due to a lack of understanding as to why the method is valid. They also lack the understanding to modify or adapt the method when faced with new and unfamiliar situations. Suggested viewing Synthetic Division: How to understand It by not doing it.

Assessing for Understanding

Students must not only be able to rewrite and divide the polynomials, but they will often need to determine the most appropriate method for performing the operation. Why questions, such as "Why did you choose inspection/long division/synthetic division to rewrite this expression?" can enhance the understanding.

Example: Express $\frac{-x^2+4x+87}{x+1}$ in the form $q(x) + \frac{r(x)}{b(x)}$.

Example: Find the quotient and remainder for the rational expression $\frac{x^3-3x^2+x-6}{x^2+2}$ and use them to write the expression in a different form.

Example: Determine the best method to simplify the following expressions, and explain why your chosen method is the most appropriate.

a) $\frac{6x^3+15x^2+12x}{3x}$ b) $\frac{x^2+9x+14}{x+7}$ c) $\frac{x^4+3x}{x^2-4}$ d) $\frac{x^3+7x^2+13x+6}{x+4}$

Instructional Resources	
Tasks	**Additional Resources**
Combined Fuel Efficiency (Illustrative Mathematics) NEW	Rational Functions Unit Classroom Task: 4.4 (Mathematics Visions Project) NEW

Back to: Table of Contents

Algebra – Arithmetic with Polynomial Expressions

NC.M3.A-APR.7a

Rewrite rational expressions.

Understand the similarities between arithmetic with rational expressions and arithmetic with rational numbers.

a. Add and subtract two rational expressions, $a(x)$ and $b(x)$, where the denominators of both $a(x)$ and $b(x)$ are linear expressions.

Concepts and Skills

Pre-requisite

- Operations with fractions
- Operations with polynomial expressions (NC.M2.A-APR.1)

Connections

- Rewrite simple rational expressions (NC.M3.A-APR.6)
- Multiple and divide rational expressions (NC.M3.A-APR.7b)
- Create and graph equations (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Justify a solution method (NC.M3.A-REI.1)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Building functions from graphs, descriptions and ordered pairs (NC.M3.F-BF.1a)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

Students should understand that the same addition and subtraction properties that apply to fractions (adding and subtracting the numerators when they have a common denominator) also apply to rational expressions. With linear denominators, greatest common factors and multiply a rational expression times 1 (constant divided by a constant) will be important points to review. In previous math classes, many students might have learned to "cross multiply" to add or subtract fractions – we must fight this misconception so students truly understand why we use a common denominator.

Note: The revised standards only have students adding and subtracting rational expressions with linear denominators, so the concept /of the common denominator can be stressed and understood, rather than more difficult algebraic manipulation.

For Example: $\frac{3x+7}{x-2} - \frac{3x+15}{2x-4}$

Assessing for Understanding

Students must be able to perform the operations and understand and explain the process (i.e. why they are factoring out a GCF, why they are finding a common denominator, why they are multiplying the numerator and denominator by the same factor, etc.)

Example: Simplify and explain your steps: $\frac{4x+13}{x-3} + \frac{x+2}{2x+6}$

Example: Why does multiplying a numerator and denominator by 2 NOT double the value of a rational expression?

Instructional Resources	
Tasks	**Additional Resources**
	Rational Functions Unit Classroom Task: 4.3 (Mathematics Visions Project) NEW

Back to: Table of Contents

Algebra – Arithmetic with Polynomial Expressions

NC.M3.A-APR.7b

Rewrite rational expressions.

Understand the similarities between arithmetic with rational expressions and arithmetic with rational numbers.

b. Multiply and divide two rational expressions.

Concepts and Skills

Pre-requisite

- Operations with fractions
- Operations with polynomial expressions (NC.M2.A-APR.1)
- Rewrite simple rational expressions (NC.M3.A-APR.6)

Connections

- Create and graph equations (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Justify a solution method (NC.M3.A-REI.1)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Building functions from graphs, descriptions and ordered pairs (NC.M3.F-BF.1a)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

Students should understand that the same multiplication and division properties that apply to fractions (multiplying the numerators and denominators for multiplication, multiplying times the reciprocal for division) also apply to rational expressions. In previous math classes, many students might have learned to "cross multiply" to divide fractions – we must fight this misconception so students truly understand why we multiply times a reciprocal.

Factoring will be a key review concept for teaching this standard.

Assessing for Understanding

Students must be able to perform the operations and understand and explain the process (i.e. why they are factoring each expression, why they can divide out common factors in the numerator and denominator, that a common denominator when dividing can be useful, etc.)

Example: Simplify and explain your steps.

a. $\left(\frac{2x+4}{x^2-6x}\right)\left(\frac{x^2-36}{4x+8}\right)$ b. $\left(\frac{x^2-4}{x^2+2x-5}\right) \div \left(\frac{x+2}{x^2+2x-5}\right)$

Instructional Resources

Tasks

Additional Resources

Rational Functions Unit Classroom Task: 4.3 (Mathematics Visions Project) NEW

Algebra – Creating Equations

NC.M3.A-CED.1

Create equations that describe numbers or relationships.

Create equations and inequalities in one variable that represent absolute value, polynomial, exponential, and rational relationships and use them to solve problems algebraically and graphically.

Concepts and Skills

Pre-requisite

- Create one variable equations and solve (NC.M2.A-CED.1)
- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Justify a solution method (NC.M3.A-REI.1)

Connections

- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Rewrite rational expressions (NC.M3.A-APR.6, NC.M3.A-APR.7a, NC.M3.A-APR.7b)
- Justify a solution method (NC.M3.A-REI.1)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11)
- Use function notation to evaluate piecewise functions (NC.M3.F-IF.2)
- Build functions from various representations and by combining functions (NC.M3.F-BF.1a, NC.M3.F-BF.1b)
- Use logarithms to express solutions to exponential equations (NC.M3.F-LE.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Student should be able to explain and defend the model they chose to represent the situation.

New Vocabulary: Absolute value equation, rational equation

Mastering the Standard

Comprehending the Standard

This is a modeling standard which means students choose and use appropriate mathematical equations to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Creating one variable equations and inequalities are included in Math 1, 2, and 3. In previous courses, students modeled with linear, exponential, quadratic, radical, and inverse

Assessing for Understanding

Students should be able to create and solve problems algebraically and graphically. There should be a focus on using methods efficiently.

Example: Clara works for a marketing company and is designing packing for a new product. The product can come in various sizes. Clara has determined that the size of the packaging can be found using the function, $p(b) = (b)(2b + 1)(b + 5)$, where b is the shortest measurement of the product. After some research, Clara determined that packaging with $20{,}500\ cm^3$ will be the most appealing to customers. What are the dimensions of the package?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
variation equations. In Math 3, students will be expected to model with polynomial, rational, absolute value, and exponential equations. Students will need to analyze a problem, determine the type of equation, and set up and solve these problems. Students may need to create an equation from different representations found in the context. This makes it important for students to realize that equations can be derived as a specific instance of an associated function. Students are expected to represent the solutions of an inequality using a number line and compound inequalities using inequality and interval notation.	**Example:** If the world population at the beginning of 2008 was 6.7 billion and growing at a rate of 1.16% each year, in what year will the population be double? **Example:** A recent poll suggests that 47% of American citizens are going to vote for the Democratic candidate for president, with a margin of error of ±4.5%. Set up and solve an absolute value inequality to determine the range of possible percentages the candidate could earn. Based on your answer, can you determine if the Democratic candidate will win the election? Why or why not? **Example:** In a Math 3 class, the red group has four members. Brian can solve an equation in 5 minutes, Luis can solve one in 4 minutes, Sylvia can solve one in 6 minutes, and Tierra can solve one in 3 minutes. Set up and solve an equation to determine how long will it take the group to complete a 10 problem worksheet if they work together. Is this answer accurate, based on the context? Why or why not?

Instructional Resources	
Tasks	**Additional Resources**
Basketball (Illustrative Mathematics)	Creating Exponential Equations

Back to: Table of Contents

Algebra – Creating Equations

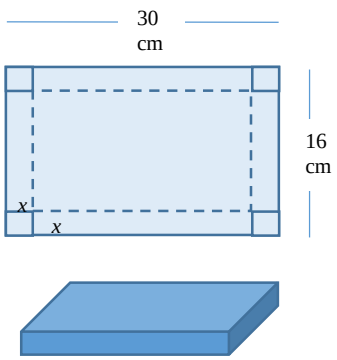
NC.M3.A-CED.2

Create equations that describe numbers or relationships.

Create and graph equations in two variables to represent absolute value, polynomial, exponential and rational relationships between quantities.

Concepts and Skills
Pre-requisite
• Create and graph two-variable equations (NC.M2.A-CED.2) • Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9) • Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b) • Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
Connections
• Understand and apply the Remainder Theorem (NC.M3.A-APR.2) • Understand the relationship between the factors of a polynomial, solutions and zeros (NC.M3.A-APR.3) • Rewrite rational expressions (NC.M3.A-APR.6, NC.M3.A-APR.7a, NC.M3.A-APR.7b) • Write the equations and inequalities of a system (NC.M3.A-CED.3) • Solve one variable rational equations (NC.M3.A-REI.2) • Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11) • Use function notation to evaluate piecewise functions (NC.M3.F-IF.2) • Analyze and compare functions (NC.M3.F-IF.7, NC.M3.F-IF.9) • Build functions from various representations and by combining functions (NC.M3.F-BF.1a, NC.M3.F-BF.1b) • Use logarithms to express solutions to exponential equations (NC.M3.F-LE.4)

The Standards for Mathematical Practices
Connections
Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard. 4 – Model with mathematics
Disciplinary Literacy
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. New Vocabulary: Absolute value equation, rational equation

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. In A-CED.1, writing and solving an equation is the essential skill required. In this standard, graphing the equation to determine key features is essential. This standard is included in Math 1, 2, and 3. Throughout all three courses, students create equations in two variables and graph them on coordinate axes. In Math 3, absolute value, polynomial, and rational graphs are introduced, and exponential graphs are further developed to solve for the exponent.	Rate of growth and decay, work rate (and other rates), geometric, and other real-world examples provide the context for many of these problems. **Example*:*** A company is manufacturing an open-top rectangular box. They have 30 cm by16 cm sheets of material. The bins are made by cutting squares the same size from each corner of a sheet, bending up the sides, and sealing the corners. Create an equation relating the volume V of the box to the length of the corner cut out x. Graph the equation and identify the dimensions of the box that will have the maximum volume. Explain. 

Instructional Resources	
Tasks	**Additional Resources**
Cockroaches (2016 Just in Time Virtual Session)	

Back to: Table of Contents

Algebra – Creating Equations

NC.M3.A-CED.3

Create equations that describe numbers or relationships.

Create systems of equations and/or inequalities to model situations in context.

Concepts and Skills

Pre-requisite

- Write the equations for a system (NC.M2.A-CED.3)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Create and graph two variable equations (NC.M3.A-CED.2)

Connections

- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11)
- Use function notation to evaluate piecewise functions (NC.M3.F-IF.2)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
1 – Make sense of problems and persevere in solving them
4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should justify the chosen models of each equation with mathematical reasoning.

Mastering the Standard

Comprehending the Standard

In Math 3, the systems of equations and inequalities that must be mastered include absolute value functions. In previous courses, students have worked with systems including linear and quadratic functions.
Function types are not limited in this standard as in previous courses. All function types are potential components of systems in Math 3. Students are not expected to solve complex systems algebraically, but should focus on more efficient method such as tables, graphs, and using technology. (Solving these systems algebraically can be an extension topic.)

Assessing for Understanding

In assessing this standard, graphical solutions can be highlighted using technology. Ideally, the functions and equations will come from a context.

Example: After receiving his business degree from UNC-Chapel Hill, John is offered positions with two companies. Company A offers him $80,000 per year, with a $1,000 increase every year. Company B offers him $60,000 per year with a 4% increase every year.

a) After how many years will the Company B salary be higher than Company A?
b) Which offer would you choose? Why?

Instructional Resources

Tasks	Additional Resources

Back to: Table of Contents

Algebra – Reasoning with Equations and Inequalities

NC.M3.A-REI.1

Understand solving equations as a process of reasoning and explain the reasoning.

Justify a solution method for equations and explain each step of the solving process using mathematical reasoning.

Concepts and Skills

Pre-requisite

- Justify a solution method and the steps in the solving process (NC.M2.A-REI.1)
- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Understand the relationship between the factors of a polynomial, solutions and zeros (NC.M3.A-APR.3)
- Rewrite rational expressions (NC.M3.A-APR.6, NC.M3.A-APR.7a, NC.M3.A-APR.7b)

Connections

- Creating one variable equations (NC.M3.A-CED.1)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Use logarithms to express solutions to exponential equations (NC.M3.F-LE.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain why it is necessary to write two equations to solve an absolute value equation.

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. In Math 3, students should extend their knowledge of all equations they are asked to solve.

When solving equations, students will use mathematical reasoning to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method.

Students do not have to use the proper names of the properties of operations and equality, but

Assessing for Understanding

Solving equations including justifications for each step, error analysis of solutions to equations, and comparing and analyzing different methods are all appropriate methods of assessing this standard.

Example: Julia is solving an absolute value inequality in class and has become stuck. Show Julia the next step and write down the explanation for that step so she can reference it on other problems.

Julia's steps:

$2|x + 5| - 3 \leq 10$

$2|x + 5| \leq 13$

$|x + 5| \leq 6.5$

Example: Describe your process for solving the following polynomial and explain the mathematical reasoning for each step.

$$x^3 + 4x^2 + x = 6$$

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
they should recognize and use the concepts associated with the properties.	**Example:** This rational equation has been solved using two different methods. **Method 1** $\frac{1}{x-8}-1=\frac{7}{x-8}$ $\frac{1}{x-8}-1\left(\frac{x-8}{x-8}\right)=\frac{7}{x-8}$ $\frac{1}{x-8}+\left(\frac{-x+8}{x-8}\right)=\frac{7}{x-8}$ $\frac{-x+9}{x-8}=\frac{7}{x-8}$ $-x+9=7$ $x=2$ **Method 2** $\frac{1}{x-8}-1=\frac{7}{x-8}$ $(x-8)\left(\frac{1}{x-8}-1\right)=\left(\frac{7}{x-8}\right)(x-8)$ $1-(x-8)=7$ $9-x=7$ $x=2$ a. Justify each step using a property of equality. b. How do the solution methods compare? c. Which method would you most likely use? Justify why.

Instructional Resources	
Tasks	**Additional Resources**

Back to: Table of Contents

Algebra – Reasoning with Equations and Inequalities

NC.M3.A-REI.2

Understand solving equations as a process of reasoning and explain the reasoning.

Solve and interpret one variable rational equations arising from a context, and explain how extraneous solutions may be produced.

Concepts and Skills

Pre-requisite

- Solve and interpret one variable inverse variation and square root equations and explain extraneous solutions (NC.M2.A-REI.2)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Rewrite rational expressions (NC.M3.A-APR.6, NC.M3.A-APR.7a, NC.M3.A-APR.7b)
- Justify a solution method and each step in the solving process (NC.M3.A-REI.1)

Connections

- Creating one variable equations (NC.M3.A-CED.1)
- Analyze and compare functions (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain when a rational equation will have an extraneous solution.

New Vocabulary: Rational equation, extraneous solution

Mastering the Standard

Comprehending the Standard

Students need to understand the process of solving rational equations, including finding the common denominator of all terms. It is important to keep in mind the limitations placed in NC.M3.A-APR.7.

Students also need to understand the relationship between rates and rational expressions, such as $peed = \frac{distance}{time}$.

Students should understand that the process of algebraically solving an equation can produce extraneous solutions. Students studied this in

Assessing for Understanding

To master this standard, students must be able to set up, solve, and evaluate the solutions to "real-world" rational equations.

Example: In a Math 3 class, the red group has four members. Brian can solve a rational equation in 5 minutes, Luis can solve one in 4 minutes, Sylvia can solve one in 6 minutes, and Tierra can solve one in 3 minutes. Set up and solve a rational equation to determine how long will it take the group to complete a 10 problem worksheet if they work together. Is this answer accurate, based on the context? Why or why not?

Additionally, students must be able to solve rational equations and understand how extraneous solutions can be produced. Graphic representations can often be used to find real solutions, but students must be able to identify when their algebraic solving process creates an extraneous solution.

Example: Consider the following equation.

$$\frac{x^2 + x - 2}{x + 2} = -2$$

Mastering the Standard

Comprehending the Standard

Math 2 in connection to square root functions. When teaching this standard, it will be important to link to the concept of having a limited domain, not only by the context of a problem, but also by the nature of the equation.

Graphically, extraneous solution can be linked to discontinuities on the graph.

Assessing for Understanding

Here are two algebraic methods that can be used to solve this equation.

Method 1:

$$\frac{x^2+x-2}{x+2} = -2$$

$$\frac{(x+2)(x-1)}{x+2} = -2$$

$$x-1 = -2$$

$$x = -1$$

Method 2:

$$\frac{x^2+x-2}{x+2} = -2$$

$$x^2+x-2 = -2\,(x+2)$$

$$x^2+x-2 = -2x-4$$

$$x^2+3x+2 = 0$$

$$(x+2)(x+1) = 0$$

$$x = -2, -1$$

Verify that each step in the two methods is correct and answer the following questions.

a) Why does Method 2 produce two solutions?

b) Looking at original equation, how can you tell which of the solutions is extraneous?

Graph the function $f(x) = \frac{x^2+x-2}{x+2}$ *on a graphing calculator or app.*

a) What do you notice about the graph?

b) Zoom into where the extraneous solution would be on the grid. What do you notice?

c) What are the implications of just looking at the graph for the solutions?

d) Now look at the table of the function. What do you notice?

Instructional Resources

Tasks	Additional Resources
	Rational Functions Unit Classroom Task: 4.7 (Mathematics Visions Project) NEW

Back to: Table of Contents

Algebra – Reasoning with Equations and Inequalities

NC.M3.A-REI.11

Represent and solve equations and inequalities graphically

Extend an understanding that the x-coordinates of the points where the graphs of two equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using a graphing technology or successive approximations with a table of values.

Concepts and Skills

Pre-requisite

- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Rewrite rational expressions (NC.M3.A-APR.6, NC.M3.A-APR.7a, NC.M3.A-APR.7b)

Connections

- Create equation to graph and solve (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.
Students should be able to explain how solutions obtained through algebraic methods and graphing can differ and understand the benefits and limitations of graphing.

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2, and 3. In previous courses, students studied linear, exponential and quadratic functions. In Math 3, the type of function is not limited. Students are expected to find a solution to any equation or system using tables, graphs and technology.

Visual examples of rational equations explore the solution as the intersection of two functions and provide evidence to discuss how extraneous solutions do not fit the model.

Assessing for Understanding

Graphical solutions, often using technology, should be highlighted in assessing student mastery of this standard.

Example*:* Graph the following system and approximate solutions for $f(x) = g(x)$.

$f(x) = \frac{x+4}{2-x}$ and $g(x) = x^3 - 6x^2 + 3x + 10$

From the standard, we build that $f(x) = g(x)$ where $f(x) = y_1$ and $g(x) = y_2$

Example*:* Use technology to solve $e^{2x} + 3x = 15$, treating each side of the statement as two equations of a system. Note: Algebraically solving equations with *e* is not an expectation of Math 3. Students should be able to solve any equations using a graphing technology.

Example: Solve the equation $5^{4x} = 2^{8x}$ graphically. Then, use the answer to show that the equation holds true for the x-value you find.

Instructional Resources

Tasks

Additional Resources

Rational Functions Unit Classroom Task: 4.7 (Mathematics Visions Project) NEW

Back to: Table of Contents

Algebra, Functions & Function Families

NC Math 1	NC Math 2	NC Math 3
Functions represented as graphs, tables or verbal descriptions in context		
Focus on comparing properties of linear function to *specific* non-linear functions and rate of change. • Linear • Exponential • Quadratic	**Focus on properties of quadratic functions and an introduction to inverse functions through the inverse relationship between quadratic and square root functions.** • Quadratic • Square Root • Inverse Variation	**A focus on more complex functions** • Exponential • Logarithm • Rational functions w/ linear denominator • Polynomial w/ degree $\leq$ three • Absolute Value and Piecewise • Intro to Trigonometric Functions
A Progression of Learning of Functions through Algebraic Reasoning		
The conceptual categories of Algebra and Functions are inter-related. Functions describe situations in which one quantity varies with another. The difference between the Function standards and the Algebra standards is that the Function standards focus more on the characteristics of functions (e.g. domain/range or max/min points), function definition, etc. whereas the Algebra standards provide the computational tools and understandings that students need to explore specific instances of functions. As students' progress through high school, the coursework with specific families of functions and algebraic manipulation evolve. Rewriting algebraic expressions to create equivalent expressions relates to how the symbolic representation can be manipulated to reveal features of the graphical representation of a function. **Note:** The Numbers conceptual category also relates to the Algebra and Functions conceptual categories. As students become more fluent with their work within particular function families, they explore more of the number system. For example, as students continue the study of quadratic equations and functions in Math 2, they begin to explore the complex solutions. Additionally, algebraic manipulation within the real number system is an important skill to creating equivalent expressions from existing functions.		

Back to: Table of Contents

NC.M3.F-IF.1

Understand the concept of a function and use function notation.

Extend the concept of a function by recognizing that trigonometric ratios are functions of angle measure.

Concepts and Skills

Pre-requisite

- Define a function (NC.M1.F-IF.1)
- Verify experimentally that the side ratios in similar triangles are properties of the angle measures in the triangle (NC.M2.G-SRT.6)
- Understand radian measure of an angle (NC.M3.F-TF.1)

Connections

- Analyze and compare functions in various representations (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build an understanding of trig functions in relation to its radian measure (NC.M3.F-TF.2a, NC.M3.F-TF.2b)
- Investigate the parameters of the sine function (NC.M3.F-TF.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the output of trig functions as unit rates.

Mastering the Standard

Comprehending the Standard

This is an extension of previous learning. Students should already understand function notation, the correspondence of inputs and outputs, and evaluating functions. In Math 3, students should build an understanding of the unique relationship between the measure of the angle and the value of the particular trig ratio.

Also in Math 3, students build an understanding of radian measure.
See NC.M3.F-TF.1 for more information.

Students should also begin to see the graphical representations of trig functions, both on a unit circle and on a graph in which the domain is the measure of the angle and the range is the value of the associated trig ratio.
On the unit circle, the input is the measure of the angle and the output of the sine function is the *y*-coordinate of the vertex of the formed triangle

Assessing for Understanding

Students should be able to create trig functions in various representations, recognizing that the domain of a trig function is the measure of the angle.

Example: Complete the function table for $f(\theta) = \sin\theta$ and $f(\theta) = \cos\theta$ and complete the following.

θ	$\sin\theta$	$\cos\theta$	θ	$\sin\theta$	$\cos\theta$
0			π		
$\frac{\pi}{6}$			$\frac{7\pi}{6}$		
$\frac{\pi}{4}$			$\frac{5\pi}{4}$		
$\frac{\pi}{3}$			$\frac{4\pi}{3}$		
$\frac{\pi}{2}$			$\frac{3\pi}{2}$		
$\frac{2\pi}{3}$			$\frac{5\pi}{3}$		
$\frac{3\pi}{4}$			$\frac{7\pi}{4}$		
$\frac{5\pi}{6}$			$\frac{11\pi}{6}$		

<u>Based on the table:</u>

a) Describe in your own words the relationship you see between the measure of the angle and the sine function.

b) If you were to graph $f(\theta) = \sin\theta$, what would it look like? What would be some of the key feature?

c) Describe in your own words the relationship between the measure of the angle and the cosine function.

d) If you were to graph $f(\theta) = \cos\theta$, what would it look like? What would be some of the key feature?

e) How does $\sin\theta$ and $\cos\theta$ relate to each other?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
and the output of the cosine function is the *x*-coordinate of the vertex of the formed triangle. See NC.M3.F-TF.2a and NC.M3.F-TF.2b for more information.	

Instructional Resources	
Tasks	**Additional Resources**

Back to: Table of Contents

Functions – Interpreting Functions

NC.M3.F-IF.2

Understand the concept of a function and use function notation.

Use function notation to evaluate piecewise defined functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Concepts and Skills

Pre-requisite

- Evaluate a function for inputs in their domain and interpret in context (NC.M1.F-IF.2)
- Interpret a function in terms of the context by relating its domain and range to its graph (NC.M1.F-IF.5)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)

Connections

- Create equation to graph and solve (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Analyze and compare functions in various representations (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able how they know a point is a solution to piecewise defined function.
New Vocabulary: piecewise function

Mastering the Standard

Comprehending the Standard

The new concept students must understand from this standard is the notation of piecewise functions – mainly, that the function must be evaluated using different function rules for the different inputs in different domains. The function rules can include the new functions for this course (polynomial, rational, exponential) and functions from previous courses (linear, quadratic, root, etc.)

Additionally, students must recognize from word problems why certain domains apply to certain function rules.

A great introduction to piecewise functions could use absolute value as a piecewise function of two linear functions. Students take a function they are learning in this course and break it into two functions they have already learned in Math 1.

Assessing for Understanding

In assessing this standard, students must be able to evaluate all types of functions, and they must be able to determine the appropriate domain to use for each input value.

Example: For the following function: $h(x) = \begin{cases} 2^x, & x < -3 \\ \frac{3}{x}, & x \geq -3 \end{cases}$

a) Evaluate $h(-4)$.
b) Evaluate $3\,h(0) + 2\,h(-3) - h(-6)$.
c) What is the domain of $h(x)$? Explain your answer.

Additionally, students must be able to explain the context of piecewise functions and how their domains apply.

Example: A cell phone company sells its monthly data plans according to the following function, with *f(x)* representing the total price and *x* representing the number of gigabytes of data used.

$$f(x) = \begin{cases} 19.95x + 60, for\ 0 \leq x \leq 3 \\ 9.95x + 75, for\ 3 < x \leq 6 \\ 125, for\ x > 6 \end{cases}$$

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
	a) If a customer uses 3 GB of data, how much will she pay? b) How many GB of data are required so a subscriber does not pay any extra money per GB? c) If you use 2.5 GB of data per month, what plan will be the cheapest? d) How many GB of monthly data will make plan B's price equal to plan C?

Instructional Resources	
Tasks	**Additional Resources**

Back to: Table of Contents

NC.M3.F-IF.4

Interpret functions that arise in applications in terms of the context.

Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities to include periodicity and discontinuities.

Concepts and Skills

Pre-requisite

- Interpret key features from graph, tables, and descriptions (NC.M2.F-IF.4)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)
- Use function notation to evaluate piecewise functions (NC.M3.F-IF.2)

Connections

- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Rewrite rational expressions (NC.M3.A-APR.6, NC.M3.A-APR.7a, NC.M3.A-APR.7b)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Analyze and compare functions (NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build functions given a graph, description or ordered pair. (NC.M3.F-BF.1a)
- Use graphs, tables and description to work with inverse functions (NC.M3.F-BF.4a, NC.M3.F-BF.4b, NC.M3.F-BF.4c)
- Use tables and graphs to understand relationships in trig functions (NC.M3.F-TF.2a, NC.M3.F-TF.2b, NC.M3.F-TF.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their identified key features with mathematical reasoning.
New Vocabulary: periodicity, discontinuity

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. Throughout all three courses, students interpret the key features of graphs and tables for a variety of different functions. In Math 3, extend to more complex functions represented by graphs and tables and focus on interpreting key features of all function types. Also, include periodicity as motion that is repeated in equal intervals of time and discontinuity as values that are not in the

Assessing for Understanding

This standard must be assessed using three important forms of displaying our functions: graphs, tables, and verbal descriptions/word problems. Students must be able to interpret each and how they apply to the key input-output values.

Example: For the function below, label and describe the key features. Include intercepts, relative max/min, intervals of increase/decrease, and end behavior.

Example: Jumper horses on carousels move up and down as the carousel spins. Suppose that the back hooves of such a horse are six inches above the floor at their lowest point and two-and-one-half feet above the floor at their highest point. Draw a graph that could represent the height of the back hooves of this carousel horse during a half-minute portion of a carousel ride.

Mastering the Standard

Comprehending the Standard

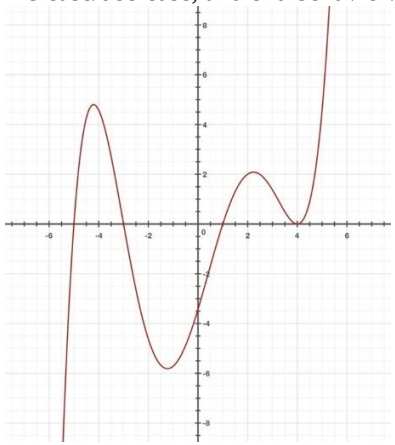
domain of a function, either as asymptotes or "holes" in the graph.

No limitations are listed with this standard. This means that all function types, even those found in more advanced courses. Students do not have to be able to algebraically manipulate a function in order to identify the key features found in graphs, tables, and verbal descriptions.
This is in contrast to NC.M3.F-IF.7, in which the specific function types are included. Students can work algebraically with those listed types and can analyze those functions in greater detail.

Students are expected to use and interpret compound inequalities using inequality and interval notation to describe key features when appropriate.

Assessing for Understanding

Example: For the function below, label and describe the key features. Include intercepts, relative max/min, intervals of increase/decrease, and end behavior.



Example: Over a year, the length of the day (the number of hours from sunrise to sunset) changes every day. The table below shows the length of day every 30 days from 12/31/97 to 3/26/99 for Boston Massachusetts.

Data on length of day

Date	12/31	1/30	3/1	3/31	4/30	5/30	6/29	7/29	8/28	9/27	10/27	11/26	12/26	1/25	2/24	3/26
Day Number	0	30	60	90	120	150	180	210	240	270	300	330	360	390	420	450
Length (hours)	9.1	9.9	11.2	12.7	14.0	15.0	15.3	14.6	13.3	11.9	10.6	9.5	9.1	9.7	11.0	12.4

During what part of the year do the days get longer? Support your claim using information provided from the table.

Instructional Resources

Tasks

Additional Resources

Rational Functions Unit Classroom Task: 4.2, 4.6, 4.7 (Mathematics Visions Project) NEW

Back to: Table of Contents

NC.M3.F-IF.7

Analyze functions using different representations.

Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.

Concepts and Skills

Pre-requisite

- Analyze functions using different representations to show key features (NC.M2.F-IF.7)
- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Write an equivalent form of an exponential expression (NC.M3.A-SSE.3c)
- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Rewrite rational expressions (NC.M3.A-APR.6, NC.M3.A-APR.7a, NC.M3.A-APR.7b)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)
- Use function notation to evaluate piecewise functions (NC.M3.F-IF.2)

Connections

- Create and graph equations in two variables (NC.M3.A-CED.2)
- Analyze graphs and tables and compare functions (NC.M3.F-IF.4, NC.M3.F-IF.9)
- Build functions (NC.M3.F-BF.1a, NC.M3.F-BF.1b)
- Understand the effects of transformations on functions (NC.M3.F-BF.3)
- Use graphs, tables and description to work with inverse functions (NC.M3.F-BF.4a, NC.M3.F-BF.4b, NC.M3.F-BF.4c)
- Compare the end behavior of functions using the rate of change (NC.M3.F-LE.3)
- Use tables and graphs to understand relationships in trig functions (NC.M3.F-TF.2a, NC.M3.F-TF.2b, NC.M3.F-TF.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should discuss which representation best shows each of the key features.

New Vocabulary: periodicity, discontinuity

Mastering the Standard

Comprehending the Standard

In previous math courses, students have identified the characteristic of graphs of other functions, including linear, quadratic, exponential, radical, and inverse variation functions. They should be familiar with the concept of intercepts, domain, range, intervals increasing/decreasing, relative maximum/minimum, and end behavior.
In Math 3, these concepts are extended to piecewise, absolute value, polynomials, exponential, rational, and sine and cosine functions. Discontinuity (asymptotes/holes) and periodicity are new features of functions that must be introduced. The intent of this standard is for students to find discontinuities in tables and graphs and to recognize their relationship to functions. Students are not expected to find an asymptote from a function. (This could be an extension topic.)

This standard will likely span multiple units, as most Math 3 courses teach polynomial, exponential, rational, and trigonometric functions in different units. These function characteristics will be repeated and reinforced throughout the course.

Students are expected to use and interpret compound inequalities using inequality and interval notation to describe key features when appropriate.

Assessing for Understanding

In assessing this standard, students must demonstrate their ability to represent and determine the key features from algebraic and graphical representations of the functions.

Example: Graph $g(x) = x^3 + 5x^2 + 2x - 8$.

a) Identify zeroes.
b) Discuss the end behavior.
c) In what intervals is the function increasing? Decreasing?

Example: Graph $y = 3\ sin\ (x) - 5$ and answer the following questions:

a) What is the period?
b) For the domain of $-2\pi < x < 2\pi$, identify any relative maxima and minima, intervals of increasing and decreasing, and lines of symmetry.

Example: For $(x) = \frac{x+4}{2-x}$, discuss end behavior and any discontinuities.

Example: Given the following piecewise function $h(x) = \begin{cases} x^2, & -3 \leq x < 3 \\ 2 - x, & 3 \leq x < 7 \end{cases}$ discuss the key features, including domain and range, intercepts, relative maximum and minimums, end behavior and discontinuities.

Instructional Resources

Tasks	Additional Resources
Running Time (Illustrative Mathematics)	Rational Functions Unit Classroom Task: 4.1, 4.2, 4.6, 4.7 (Mathematics Visions Project) NEW Polynomial Functions Unit Classroom Task: 3.3 (Mathematics Visions Project) NEW

Back to: Table of Contents

Functions – Interpreting Functions

NC.M3.F-IF.9

Analyze functions using different representations.

Compare key features of two functions using different representations by comparing properties of two different functions, each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

Concepts and Skills

Pre-requisite

- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

Connections

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should discuss how the comparison of a functions leads to a mathematical understanding, such as with transformations and choosing better models.
New Vocabulary: periodicity, discontinuity

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. Throughout all three courses, students compare properties of two functions. The representations of the functions should vary: table, graph, algebraically, or verbal description.

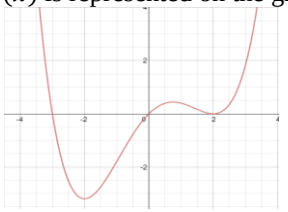
In Math 3, this standard can include two functions of any type students have learned in high school math in any representation. Comparing the key features should be the focus of the teaching for this standard, so the actual functions involved are not as important.

Students are expected to use and interpret compound inequalities using inequality and interval notation to describe key features when appropriate.

Assessing for Understanding

In assessing this standard, students must demonstrate that they can not only identify, but compare, the key features of two different functions. Appropriate question stems could include: Which is less/greater; Which will have a greater value at x = __; Which function has the higher maximum/lower minimum; etc.

Examples: If $f(x) = -(x+7)^2(x-2)$ and $g(x)$ is represented on the graph.



a. What is the difference between the zero with the least value of $f(x)$ and the zero with the least value of $g(x)$?
b. Which has the largest relative maximum?
c. Describe their end behaviors. Why are they different? What can be said about each function?

Mastering the Standard

Comprehending the Standard

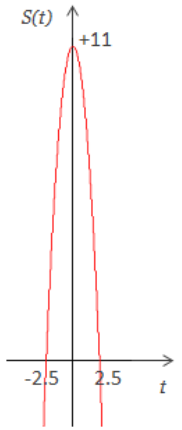
Assessing for Understanding

Example: Two objects dropped downward at the same time from a top of building. For both functions, *t* represents seconds and the height is represented in feet.

The function's data of the first object is given by this table:

t	*s*(*t*)
0	20
2.5	15
3.5	10
4.3	5
5	0

The function's graph of the second object is shown at the right:



a) Which object was dropped from a greater height? Explain your answer.
b) Which object hit the ground first? Explain your answer.
c) Which object fell at a faster rate (in ft/sec)? Explain your answer.

Instructional Resources

Tasks

Additional Resources

Polynomial Functions Unit Classroom Task: 3.3 (Mathematics Visions Project) NEW

Back to: Table of Contents

NC.M3.F-BF.1a

Build a function that models a relationship between two quantities.

Write a function that describes a relationship between two quantities.

a. Build polynomial and exponential functions with real solution(s) given a graph, a description of a relationship, or ordered pairs (include reading these from a table).

Concepts and Skills

Pre-requisite

- Build quadratic functions given a graph, description, or ordered pair (NC.M2.F-BF.1)
- Create equation to graph and solve (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

Connections

- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Write an equivalent form of an exponential expression (NC.M3.A-SSE.3c)
- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Understand the effects of transforming functions (NC.M3.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss when multiple models can describe the information given, for example, when given the two roots, multiple models can contain those roots.

Mastering the Standard

Comprehending the Standard

This standard relates to building functions in two different contexts – polynomial (with real solutions) and exponential. In many Math 3 courses, it will be covered in two different units.

When building polynomial functions, only those with real solutions are considered. The relationship between solutions and factors, multiplicity and graphs, and the leading coefficient's sign relating to the end behaviors are all essential to build these functions.

When building exponential functions, students must be able to determine the initial value (a) and rate of change (b) from the table, graph, or

Assessing for Understanding

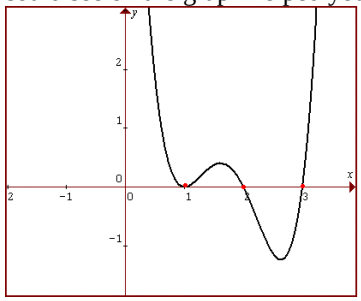
For both functions, it is important that the assessment questions include algebraic "math" questions and questions in context. The answers to questions assessing this standard should be the actual function they are building, as other standards allow students to identify and interpret key features.

Example: Build polynomial functions with a double root at -2 and another root at 5.
This example should be connected to NC.M3.F-BF.3, as students should understand which transformations functions do not change the zeros of the functions. This could also be connected to NC.M3.N-CN.9, as students should understand how to create multiple equations that could be solved with the same roots.

Example: The population of a certain animal being researched by environmentalists has been decreasing substantially. Biologists tracking the species have determined the following data set to represent the remaining animals:

Year	2010	2011	2012	2013	2014
Pop.	40,000	30,000	22,500	16.875	12,656

Assuming the population continues at the same rate, what function would represent the population $f(x)$ in year x, assuming x is the number of years after the year 2000?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
description presented. These problems can include those with compounding interest, continuous relationships involving *e*, and doubling time/half-life.	**Example**: Build a polynomial function that could represent the following graph, and explain how each characteristic you could see on the graph helped you build the function. 

Instructional Resources	
Tasks	**Additional Resources**
Cockroaches (2016 Just In Time Virtual Session) NEW	Polynomial Functions Unit Classroom Task: 3.1 (Mathematics Visions Project) NEW Truncated Graph NEW

Back to: Table of Contents

NC.M3.F-BF.1b

Build a function that models a relationship between two quantities.

Write a function that describes a relationship between two quantities.

b. Build a new function, in terms of a context, by combining standard function types using arithmetic operations.

Concepts and Skills

Pre-requisite

- Build new function by combine linear, quadratic and exponential functions (NC.M1.F-BF.1b)
- Operations with polynomials (NC.M1.A-APR.1)
- Operations with rational expressions (NC.M3.A-APR.7a, NC.M3.A-APR.7b)

Connections

- Create equation to graph and solve (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify new function and discuss how the new function fits the context.

Mastering the Standard

Comprehending the Standard

This standard asks students to combine standard function types by addition, subtraction, and multiplication. In Math 3, we are NOT required to include composition, although it could be a valuable extension.

The key concept for teaching this standard is a review of adding and subtracting expressions (including combining like terms) and multiplying expressions (distributing polynomials and exponent rules).

Assessing for Understanding

In assessing this standard, students will need to perform the operations and determine from a context which operation is appropriate. The functions that students need to combine should be given in problems, but the operation can be determined from context if necessary.

Example: Last year, army engineers modeled the function of a bullet fired by a United States soldier from a certain weapon. The function f(x) $= -16x^2 + 200x + 4$ modeled the path of the bullet. This year, the soldiers were supplied with more powerful guns that changed the path of the bullet from higher ground by adding the function $g(x) = 300x + 20$. What function models the path of the new bullet?

Example: Consider the functions: $f(x) = 4x + 9$ and $g(x) = -2x - 4$

a) Evaluate $f(-3)$.
b) Evaluate $g(-3)$.
c) Add $f(x) + g(x)$.
d) Evaluate $(f + g)(-3)$.
e) What do you notice? What properties have you learned that explain your answer?

Mastering the Standard	
	Example: A cup of coffee is initially at a temperature of 93º F. The difference between its temperature and the room temperature of 68º F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time.

Instructional Resources	
Tasks	**Additional Resources**
	Polynomial Functions Unit Classroom Task: 3.1, 3.4 (Mathematics Visions Project) NEW

Back to: Table of Contents

Functions – Building Functions

NC.M3.F-BF.3

Build new functions from existing functions.

Extend an understanding of the effects on the graphical and tabular representations of a function when replacing $f(x)$ with $k \cdot f(x)$, $f(x) + k$, $f(x + k)$ to include $f(k \cdot x)$ for specific values of k (both positive and negative).

Concepts and Skills

Pre-requisite

- Understand the effects of transformations on functions (NC.M2.F-BF.3)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)

Connections

- Analyze and compare the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build polynomial and exponential functions from a graph, description, or ordered pairs (NC.M3.F-BF.1a)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct a viable argument and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain why $f(x + k)$ moves the graph of the function left or right depending on the value of *k*.

Mastering the Standard

Comprehending the Standard

Students learned the translation and dilation rules in Math 2 with regard to linear, quadratic, square root, and inverse variation functions. In Math 3, we apply these rules to functions in general.

Students should conceptually understand the transformations of functions and refrain from blindly memorizing patterns of functions. Students should be able to explain why $f(x + k)$ moves the graph of the function left or right depending on the value of *k*.

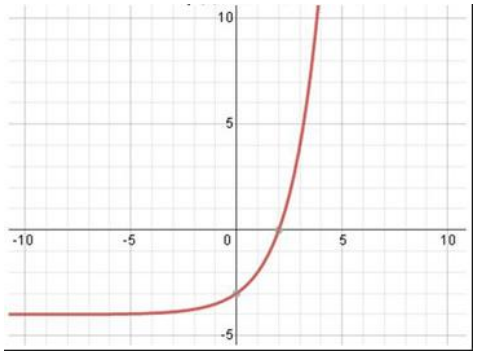
• Note: Phase shifts and transformations of trigonometric functions are NOT required in Math 3. Those will be covered in the fourth math course.

Assessing for Understanding

In demonstrating their understanding, students must be able to relate the algebraic equations, graphs, and tabular representations (ordered pairs) as functions are transformed. Appropriate questions will ask students to identify and explain these transformations.

Example: The graph of $f(x)$ and the equation of $g(x)$ are shown below. Which has a higher y-intercept? Explain your answer.

$f(x)$:



$g(x) = 2^x - 7$

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
	Example: Use the table below to identify the transformations and write the equation of the absolute value function f(x).

x	-6	-5	-4	-3	-2
f(x)	3	1	-1	1	3

Example: Why does $g(x) = \frac{1}{x-3}$ shift to the right three units from the rational function $f(x) = \frac{1}{x}$?

Instructional Resources	
Tasks	**Additional Resources**
Transforming the Graph of a Function (Illustrative Mathematics)	Rational Functions Unit Classroom Task: 4.2 (Mathematics Visions Project) NEW

Back to: Table of Contents

NC.M3.F-BF.4a

Build new functions from existing functions.

Find an inverse function.

a. Understand the inverse relationship between exponential and logarithmic, quadratic and square root, and linear to linear functions and use this relationship to solve problems using tables, graphs, and equations.

Concepts and Skills

Pre-requisite

- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

Connections

- The existence of an inverse function and representing it (NC.M3.F-BF.4b, NC.M3.F-BF.4c)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the relationship between inverse operations and inverse functions.
New Vocabulary: inverse function

Mastering the Standard

Comprehending the Standard

Students have used inverse operations to solve equations in previous math courses, but this is the first time students are introduced to the concept of an inverse function. All of the F-BF.4 standards relate, but the progression of understanding the relationship, determining is an inverse exists, and solving for the inverse through the F-BF.4a, F-BF.4b, and F-BF.4c will enhance understanding.

For this part of the standard, the main concept students must understand is that an inverse function switches the input and output (x and y) for every point in the function. It is important to connect this concept to the reflection of one function, $f(x)$, across the line of symmetry $y = x$, to create the inverse function, $g(x)$. In Math 3, we are limiting the functions to linear, quadratic, square root, exponential, and logarithmic.

Assessing for Understanding

Students should first start by exploring the relationships between inverse functions.

Example: Complete the following tables for the given functions. Which are inverses? Explain your answer.

$f(x) = \frac{1}{10}x$

X	0	1	2	3	4
f(x)					

$g(x) = 10^x$

X	0	1	2	3	4
f(x)					

$h(x) = 10x$

X	0	1	2	3	4
f(x)					

Mastering the Standard

Comprehending the Standard

Students must also understand the common notation f^{-1} to represent inverse functions.

Students, while having worked with quadratic and square root functions, may not have explored all aspects of the inverse relationship.

Students started work with exponential functions in NC Math 1, and have not been exposed to logarithms before this course.

When speaking of inverse relationships, it is important for students to understand and communicate the reasoning for finding an inverse function. This can often be accomplished by considering the independent and dependent variables, the context of the problem, and a chosen solution pathway.

Assessing for Understanding

$j(x) = \log_{10} x$

X	1	100	1,000	10,000	100,000
f(x)					

As students are solving problems using inverses, common formulas can help students understand this inverse relationship (Celsius/Fahrenheit conversions, geometry formulas, interest formulas). To understand the concept of an inverse function, students should be asked to explain the input as a function of the output and how this affects the values.

Example: The area of a square can be described as a function of the length of a side, $A(s) = s^2$.
What is the area of a square with side length 5 cm?
What is the length of a side of a square with an area 25 cm^2?
What relationship do a function of area given a side length and a function of side length given the area share? How do you know?
Use this relationship to solve for the length of a side of a square with an area of 200 cm^2.

Example: Complete the table to write the inverse for the following function. Is the inverse a function? Explain your answer.

x	1	2	3	4	5
f(x)	2	4	9	4	12

x					
f^{-1} (x)					

Instructional Resources

Tasks	Additional Resources
Water Tower Task (2016 Summer Information Session) Cockroaches (2016 Just in Time Virtual Session)	Inverse Graphing Discovery

Back to: Table of Contents

NC.M3.F-BF.4b

Build new functions from existing functions.

Find an inverse function.

b. Determine if an inverse function exists by analyzing tables, graphs, and equations.

Concepts and Skills

Pre-requisite

- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)
- Understand inverse relationships (NC.M3.F-BF.4a)

Connections

- Represent inverse functions (NC.M3.F-BF.4c)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the reasoning in needing a restricted domain.
New Vocabulary: inverse function

Mastering the Standard

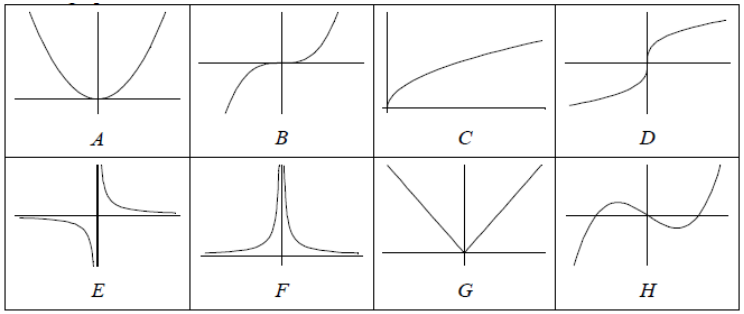
Comprehending the Standard

In Math 1, students learned to determine if a relation is a function by analyzing tables, equations, and graphs. In Math 3, students need to determine if a function is invertible and on what domain.
This part of the standard is not limited by function type. This means that students should be able to determine if any function or a portion of the function has an inverse function from different representations.

Assessing for Understanding

The standard states that students must determine if an inverse function exists, so presenting graphs, tables, and equations are all appropriate representations for students to analyze. Additionally, especially for quadratic functions, students must be able to determine the appropriate domain for a function to have an inverse.

Example: Which of the following functions have inverse functions? For those that are do not have inverse functions as a whole, divide the graph into sections that do have inverse functions.



Example: Use a table of $f(x) = 3x^2 - 18x + 5$ to determine possible domains on which $f^{-1}(x)$ is a function.

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
	Example: Which of the following equations have an inverse function? How do you know, from the table and graph? For any that do not, how can we limit the domain of the function to ensure that it has an inverse? a) $f(x) = 2x$ b) $f(x) = x^2$ c) $f(x) = 2^x$

Instructional Resources	
Tasks	**Additional Resources**

Back to: Table of Contents

NC.M3.F-BF.4c

Build new functions from existing functions.

Find an inverse function.

c. If an inverse function exists for a linear, quadratic and/or exponential function, f, represent the inverse function, f^{-1}, with a table, graph, or equation and use it to solve problems in terms of a context.

Concepts and Skills

Pre-requisite

- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)
- Understand inverse relationships and determine if an inverse exist (NC.M3.F-BF.4a, NC.M3.F-BF.4b)

Connections

- Use logarithms to expression solutions to exponential functions (NC.M3.F-LE.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should discuss which representation (tabular, graphical, or symbolic) is the most efficient to solve a particular problem.
New Vocabulary: inverse function

Mastering the Standard

Comprehending the Standard

Once students understand the concept of a function that has an inverse, they can begin solving for the inverse functions. The idea of reversing the input and output (x and y) is central to solving for an inverse algebraically, and it should also be emphasized on the graph (reflection over the $y = x$ line) and table.
It is important to note; the algebraic approach can be complex in many cases. Often, tables and graphs can be used to solve problems in a more efficient and student friendly manner.

In Math 3, the functions are limited to linear, quadratic, and exponential. For quadratics, it must be emphasized that we have the equation in a form we can solve for the input variable, so this can be an appropriate concept in which to review completing the square and vertex form, from Math 2.

Assessing for Understanding

Most assessment items for this standard will ask students to solve for an inverse using a graph or equation. Real-world context exists with common conversion formulas, area/volume formulas, and interest formulas.

Example*:* Graph the inverse of $f(x) = -\frac{3}{2}x - 3$. How does $f^{-1}(x)$ relate to $f(x)$?

Example*:* Find the inverse of the function $g(x) = 2^x$ and demonstrate it as the inverse using input – output pairs.

Example*:* Let $h(x) = x^3$. Find the inverse function.

Example: Let $f(x) = x^2 + 7x + 9$. Does an inverse function exist for the entire domain of the function? Find the inverse of this function.

Instructional Resources	
Tasks	**Additional Resources**
Cockroaches (2016 Just In Time Virtual Session)	

Back to: Table of Contents

NC.M3.F-LE.3

Construct and compare linear and exponential models and solve problems.

Compare the end behavior of functions using their rates of change over intervals of the same length to show that a quantity increasing exponentially eventually exceeds a quantity increasing as a polynomial function.

Concepts and Skills

Pre-requisite

- Calculate and interpret the average rate of change (NC.F-IF.6)
- Compare the end behavior of linear, exponential and quadratic functions (NC.M1.F-LE.3)
- Analyze and compare the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.7, NC.M3.F-IF.9)

Connections

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the rate of change for each function type as the value of the domain increases.

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1 and 3. In previous courses, students studied linear, exponential, and quadratic models. In Math 3, polynomial functions are included.

For Example: For the functions $f(x) = x^3$ and $g(x) = 3^x$, which function has a greater value at:

a) $x = 0.5$ b) $x = 1$ c) $x = 1.5$ d) $x = 2$
e) $x = 2.5$ f) $x = 3$ g) $x = 3.5$ h) $x = 4$

Assessing for Understanding

Students must demonstrate that they understand how exponential functions ultimately increase at a greater rate than polynomial functions when considering the end behavior – namely, the rate of change is greater for an exponential function as the function increases to infinity.

Example: Using technology, determine the average rate of change of the following functions for intervals of their domains in the table.

Functions	Average rate of change $0 \le x \le 10$	Average rate of change $10 \le x \le 20$	Average rate of change $20 \le x \le 30$	Average rate of change $30 \le x \le 40$	Average rate of change $40 \le x \le 50$
$f(x) = x^3$					
$f(x) = 1.3^x$					

a) When does the average rate of change of the exponential function exceed the average rate of change of the polynomial function?

b) Using a graphing technology, graph both functions. How do the average rates of change in your table relate to what you see on the graph?

Note: You can use the information in your table to determine how to change the setting to see where the functions intersect.

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
	c) In your graphing technology, change the first function to $f(x) = x^4$ and adjust the settings to see where the functions intersect. What do you notice about the rates of change interpreted from the graph? d) Make a hypothesis about the rates of change about polynomial and exponential function. Try other values for the exponent of the polynomial function to support your hypothesis.

Instructional Resources	
Tasks	**Additional Resources**
	Polynomial Functions Unit Classroom Task: 3.2 (Mathematics Visions Project) NEW

Back to: Table of Contents

NC.M3.F-LE.4

Construct and compare linear and exponential models and solve problems.

Use logarithms to express the solution to $ab^{ct} = d$ where a, b, c, and d are numbers and evaluate the logarithm using technology.

Concepts and Skills

Pre-requisite

- Create equation to graph and solve (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Justify a solution method and each step in the solving process (NC.M3.A-REI.1)
- Understand the inverse relationship between functions (NC.M3.F-BF.4a)
- Represent inverse functions (NC.M3.F-BF.4c)

Connections

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss logarithms as the inverse function of an exponential function.
New Vocabulary: logarithm

Mastering the Standard

Comprehending the Standard

Building on the inverse relationship students conceptualized for exponents and logarithms in F-BF.4, students will rewrite exponents in logarithmic form and use it to solve equations, both algebraically and in the context of word problems.

Students will also need to be able to determine numerical approximations for the logarithms using technology.
For Example: Rewrite the following in logarithmic form. Then, evaluate the logarithms using technology.
$a)\ 10^x = 1000$ b) $3^x = 1000$

Students should use the relationship between exponential and logarithmic functions to solve problems.

$$b^c = d \leftrightarrow \log_b d = c$$

a. Students can use substitution to reveal another relationship that can be used to solve the original problem. For example:

$$5^{x+3} = 372$$

Assessing for Understanding

Students must demonstrate the ability to solve exponential equations for an exponent variable using logarithms, and they should be able to express their answer in logarithmic form and using a decimal approximation.

Example: Consider the following investments.

a) A parent invests $2,000 at a 5% interest rate to help his daughter save for college. How long will it take his money to double? (Show your equation and the work.)
b) A banker invests $50,000 at a 5% interest rate to make money for Wells Fargo. How long will it take the bank's money to double? (Show your equation and the work.)
c) What do you notice about the answers? Based on your work, why is that the case?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
The goal is to rewrite each expression so they both have the same base. In this case, we are using 10. Starting with the expression on the left, $5 = 10^m$, rewrite using logarithmic form. We see that $m = \log_{10} 5$. Using substitution, this means that $5 = 10^{\log_{10} 5}$ Using the same procedure with the expression on the right we get, $372 = 10^{\log_{10} 372}$. We can now substitute these back into the original equation. $$5^{x+3} = 372$$ $$\left(10^{\log_{10} 5}\right)^{x+3} = 10^{\log_{10} 372}$$ Because this is an equation and both sides of the equation are base 10, the exponents must be equal. This reveals a new equation that can be used to solve for x. $$(\log_{10} 5)(x+3) = \log_{10} 372$$ $$x = \frac{\log_{10} 372}{\log_{10} 5} - 3$$ $$x \approx .6776$$ b. Students are expected to rewrite an exponential equation into logarithmic form to find or approximate a solution. For example: $$5^{x+3} = 372$$ $$\log_5 372 = x + 3$$ $$\log_5 372 - 3 = x$$ $$x \approx .6776$$ Students are **not** expected to know or use the properties of logarithms, *e*, or natural logs to solve problems. These can be extension topics, but are beyond the scope of the NC Math 3 standards.	

Instructional Resources	
Tasks	**Additional Resources**
Cockroaches (2016 Just In Time Virtual Session)	

Back to: Table of Contents

NC.M3.F-TF.1

Extend the domain of trigonometric functions using the unit circle.

Understand radian measure of an angle as:

- The ratio of the length of an arc on a circle subtended by the angle to its radius.
- A dimensionless measure of length defined by the quotient of arc length and radius that is a real number.
- The domain for trigonometric functions.

Concepts and Skills

Pre-requisite

- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)

Connections

- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)
- Define radian measure (NC.M3.G-C.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the relationship between degrees and radians.
New Vocabulary: arc length

Mastering the Standard

Comprehending the Standard

To build the understanding of radian measure, students should first become familiar with degree measure.

In ancient times, when discussing angle measure, it was realized that the best way to describe angle measure was through a ratio. It was decided based on a different numbering system that they would divide a circle into 360 sectors and each of the sectors would measure 1 degree. The division of the circle into 360 sectors not only divided the angle, but also divided the arc of the circle as well. (Hence the measure of the central angle is the same as the measure of the intercepted arc.)

This means that a measure of 42° is 42 $\left(\frac{1}{360}\right)$ of a circle or 42 divisions of the 360 divisions.

In modern times, as science and mathematics knowledge increased, the decision to divide a circle into 360 parts is arbitrary and less precise. This lead to the development of radian measures.

In this process, a ratio is still used, however the circle is not divided into parts but is described in the ratio of the circumference to the radius.

Here is a good resource to understand radian measure: Find radian measure by dividing arc length by radius (Learn Zillion)

Assessing for Understanding

In mastering this standard, students will need to demonstrate an understanding of radian angle measure and applying the arc length formula (Arc Length = Radius • Radian Measure) to solve for any missing measure, both using basic measures and in the context of word problems. They following examples are from NC.M3.G-C.5 but require the understanding of this standard.

Example: An angle with a measure of 4 radians intercepts an arc with a length of 18 ft. What is the length of the radius of the circle?

Example*:* The minute hand on the clock at the City Hall clock in Stratford measures 2.2 meters from the tip to the axle.

a) Through what radian angle measure does the minute hand pass between 7:07 a.m. and 7:43 a.m.?
b) What distance does the tip of the minute hand travel during this period?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
By discovery (using string, rolling a can, etc.), students can determine that it takes just over 6 radii to create the circumference of a circle, and the teacher can relate that to 2π.	

Instructional Resources	
Tasks	**Additional Resources**
	Converting Degrees and Radians Trigonometric Functions Unit Classroom Task: 6.6, 6.7, 6.8, 6.9 (Mathematics Visions Project) expectations NEW

Back to: Table of Contents

NC.M3.F-TF.2a

Extend the domain of trigonometric functions using the unit circle.

Build an understanding of trigonometric functions by using tables, graphs and technology to represent the cosine and sine functions.

a. Interpret the sine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its *y* coordinate.

Concepts and Skills

Pre-requisite

- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)
- Understand radian measure (NC.M3.F-TF.1)

Connections

- Analyze and compare the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should describe the relationship between sine represented on a unit circle and graphical representation of the sine function.

Mastering the Standard

Comprehending the Standard

Students will be introduced to the unit circle and angle measures on the coordinate plane in Math 3 as a way to relate the sine and cosine ratios to the coordinates and the plane.

A unit circle is used to develop the concepts of this standard to simplify the picture for students. In Math 3, students are only introduced to the trigonometric functions.

This standard builds upon previous understanding of the trig ratios in right triangles.

Sin θ is the unit rate produced by the ratio of the length of the opposite side to the length of the hypotenuse.

Assessing for Understanding

Students apply reasoning to their knowledge of the relationship between angles and the sides of right triangles.

Example: A stink bug has crawled into a box fan and sits on the tip of the blade of the fan as seen below. The fan starts to turn slowly due to a breeze in the room.

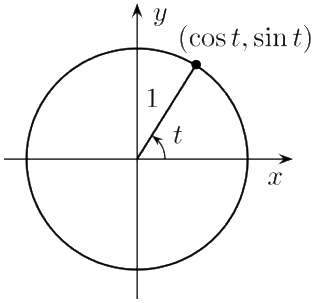
a) Create a function and a graph that describes its change in height from its original position based on the angle of the blade from its original position.

b) What is the height of the stink bug when the blade has rotated 2 radians? $\frac{11\pi}{6}$ radians?

c) How much has the blade rotated when the stink bug's height is $-\frac{3}{4}$ feet? Can there be more than one answer?

Mastering the Standard

Comprehending the Standard



$$\sin\theta = \frac{length\ of\ opposide\ side}{length\ of\ hypotenuse}$$

Since we are working within a unit circle, and the hypotenuse is the radius of the unit circle, so the length of the hypotenuse is 1 unit. This means that $\sin\theta = \frac{length\ of\ opposide\ side}{1}$, so with the unit circle, $\sin\theta$ is the length of the opposite side.
This means that the height of the triangle, which is the y-coordinate of the vertex on the circle, is $\sin\theta$.

The focus of this standard is on the relationship between the changing angle of the sine function and the value of the sine ratio. This should allow students to move from the unit circle to graphing the relationship on a coordinate plane in which the independent variable is the angle measure and the dependent variable is the value of the sine ratio (the y-coordinate from the unit circle). This is a strong connection to NC.M3.F-IF.1.

In general, from the unit circle, students should see that as the angle is near zero, the ratio of the length of the opposite side to the length of the hypotenuse is also near zero. As the angle starts to increase and approaches 90° or $\frac{\pi}{2}$, the value of the sine ratio approaches 1. This pattern continues around the unit circle and eventually demonstrates the periodicity of the sine function.

An in depth teaching of the unit circle, tangent and reciprocal ratios, coterminal angles, specific coordinates and the Pythagorean Identity are NOT appropriate for Math 3, as they will be covered in depth in the fourth math course.

Students should understand these relationships in degree and radian angle measure.

Assessing for Understanding

Instructional Resources

Tasks

Additional Resources

Trigonometric Functions Unit Classroom Task: 6.3, 6.6, 6.7, 6.8, 6.9 (Mathematics Visions Project) NEW

Back to: Table of Contents

NC.M3.F-TF.2b

Extend the domain of trigonometric functions using the unit circle.

Build an understanding of trigonometric functions by using tables, graphs and technology to represent the cosine and sine functions.

b. Interpret the cosine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its *x* coordinate.

Concepts and Skills

Pre-requisite

- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)
- Understand radian measure (NC.M3.F-TF.1)

Connections

- Analyze and compare the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should describe the relationship between cosine represented on a unit circle and graphical representation of the cosine function.

Mastering the Standard

Comprehending the Standard

Students will be introduced to the unit circle and angle measures on the coordinate plan in Math 3 as a way to relate the sine and cosine ratios to the coordinates and the plane.

A unit circle is used to develop the concepts of this standard to simplify the picture for students. In Math 3, students are only introduced to the trigonometric functions.

This standard builds upon previous understanding of the trig relationship in right triangle. $\cos\theta$ is the unit rate produced by the ratio of the length of the adjacent side to the length of the hypotenuse.

Assessing for Understanding

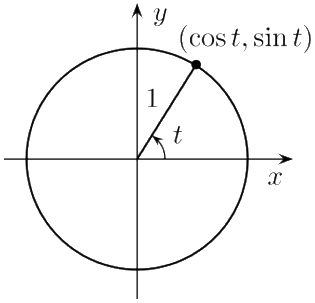
Students apply reasoning to their knowledge of the relationship between angles and the sides of right triangles.

As the angle changes, sine represents the change in the y-coordinate (height of the triangle) on the unit circle, cosine represents the change in the x-coordinate (length of the base of the unit circle).

Students should be able to not only see the relationship between the functions represented on a unit circle and the graphical representation on the coordinate plane, but should understand the relationship between the sine and cosine functions.

Mastering the Standard

Comprehending the Standard



$\cos\theta = \frac{length\ of\ adjacent\ side}{length\ of\ hypotenuse}$

Since we are working within a unit circle, and the hypotenuse is the radius of the unit circle, so the length of the hypotenuse is 1 unit. This means that $\cos\theta = \frac{length\ of\ adjacent\ side}{1}$, so with the unit circle, $\cos\theta$ is the length of the adjacent side.
This means that the base of the triangle, which is the *x*-coordinate of the vertex on the circle, is $\cos\theta$.
The focus of this standard is on the relationship between the changing angle of the cosine function and the value of the cosine ratio. This should allow students to move from the unit circle to graphing the relationship on a coordinate plane in which the independent variable is the angle measure and the dependent variable is the value of the cosine ratio (the *x*-coordinate from the unit circle). This is a strong connection to NC.M3.F-IF.1. From the unit circle, students should see that as the angle is near zero, the ratio of the length of the opposite side to the length of the hypotenuse is also near 1. As the angle starts to increase and approaches 90° or $\frac{\pi}{2}$, the value of the cosine ratio approaches 0. This pattern continues around the unit circle and eventually demonstrates the periodicity of the cosine function.

An in depth teaching of the unit circle, tangent and reciprocal ratios, coterminal angles, specific coordinates and the Pythagorean Identity are NOT appropriate for Math 3, as they will be covered in depth in the fourth math course.

Students should understand these relationships in degree and radian angle measure.

Assessing for Understanding

Instructional Resources

Tasks

Additional Resources

Trigonometric Functions Unit Classroom Task: 6.3, 6.6, 6.7, 6.8, 6.9 (Mathematics Visions Project) NEW

Back to: Table of Contents

Functions – Trigonometric Functions

NC.M3.F-TF.5

Model periodic phenomena with trigonometric functions.

Use technology to investigate the parameters, a, b, and h of a sine function, $f(x) = a \cdot sin(b \cdot x) + h$, to represent periodic phenomena and interpret key features in terms of a context.

Concepts and Skills

Pre-requisite

- Interpret parts of an expression in context (NC.M3.A-SSE.1a)
- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)
- Understand radian measure (NC.M3.F-TF.1)
- Build an understanding of trig functions (NC.M3.F-TF.2a, NC.M3.F-TF.2b)

Connections

- Analyze and compare the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss how changing the parameters effects the different representations.
New Vocabulary: period, amplitude

Mastering the Standard

Comprehending the Standard

It is important to not overreach with this standard. In Math 3, students are just being introduced to the concepts of the sine function and the effects of the various representations by changing parameters.
As the phrase at the beginning of the standards states, students should use technology to investigate these changes.

There are several excellent online resources to investigate the change in parameters of trig functions. For some of these resources, you may need to create an account. Some of these resources are listed below. Some of the resources explore horizontal phase shift, which is not part of this standard.

Phase shifts and complicated trigonometric functions are not part of the standards for Math 3, as they will be covered in depth in the fourth math course. This is an introduction to the concept of a periodic graph through learning the sine function.

Assessing for Understanding

Students should be able to explain how the change in parameters effects the various representations and interpret them in a context.

Example: The following function describes the stock price for Facebook where m stands for the number of months since May 2012. Use technology to graph and create tables as needed.

$$f(m) = -11 \sin\left(\frac{2\pi}{4}m\right) + 38$$

a) Interpret the 38 in the context of the problem.
b) What does -11 mean in context of the problem and what is the significance of 11 being negative?
c) How long does it take for the pattern to start repeating?
d) During which months would you want to buy and sell stock in Facebook?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**

Instructional Resources	
Tasks	**Additional Resources**
Representing Trigonometric Functions NEW	Graphing the Sine Function using Amplitude, Period, and Vertical Translation (Desmos.com) A visual explanation of the characteristics of the Sine Function (Geogebra.org) Trigonometric Functions Unit Classroom Task: 6.1, 6.2, 6.4, 6.10, 6.11, 6.12 (Mathematics Visions Project) NEW

Back to: Table of Contents

Geometry

NC Math 1	NC Math 2	NC Math 3
Analytic & Euclidean		
Focus on coordinate geometry • Distance on the coordinate plane • Midpoint of line segments • Slopes of parallel and perpendicular lines • Prove geometric theorems algebraically	**Focus on triangles** • Congruence • Similarity • Right triangle trigonometry ○ Special right triangles	**Focus on circles and continuing the work with triangles** • Introduce the concept of radian • Angles and segments in circles • Centers of triangles • Parallelograms
A Progression of Learning		
Integration of Algebra and Geometry • Building off of what students know from 5^{th} – 8^{th} grade with work in the coordinate plane, the Pythagorean theorem and functions. • Students will integrate the work of algebra and functions to prove geometric theorems algebraically. • Algebraic reasoning as a means of proof will help students to build a foundation to prepare them for further work with geometric proofs.	**Geometric proof and SMP3** • An extension of transformational geometry concepts, lines, angles, and triangles from 7^{th} and 8^{th} grade mathematics. • Connecting proportional reasoning from 7^{th} grade to work with right triangle trigonometry. • Students should use geometric reasoning to prove theorems related to lines, angles, and triangles. *It is important to note that proofs here are not limited to the traditional two-column proof. Paragraph, flow proofs and other forms of argumentation should be encouraged.*	**Geometric Modeling** • Connecting analytic geometry, algebra, functions, and geometric measurement to modeling. • Building from the study of triangles in Math 2, students will verify the properties of the centers of triangles and parallelograms.

Back to: Table of Contents

NC.M3.G-CO.10

Prove geometric theorems.

Verify experimentally properties of the centers of triangles (centroid, incenter, and circumcenter).

Concepts and Skills

Pre-requisite

- Use triangle congruence to prove theorems about lines, angles, and segments in triangles (NC.M2.G-CO.10)

Connections

- Understand and apply theorems about circles (NC.M3.G-C.2)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: centroid, incenter, circumcenter

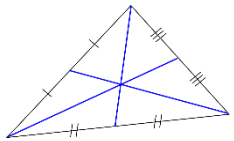
Mastering the Standard

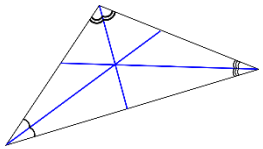
Comprehending the Standard

The goal is for students to be able to explore, make conjectures about the intersection of the different straight objects that produce the triangle centers, to justify why all three straight objects intersect at a common point, and why that point is an important feature of the triangle. The centers of triangles should be explored dynamically where students can discover them and their properties.

The centers of triangles are also known as points of concurrency for triangles. The three centers that are a focus for Math 3 are:

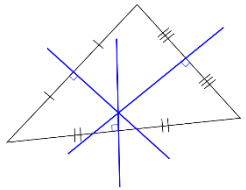
- Centroid – the point where the three medians of a triangle intersect





- Incenter – the point where the three angle bisectors of a triangle intersect

- Circumcenter – the point where the three perpendicular bisectors of the sides of a triangle intersect

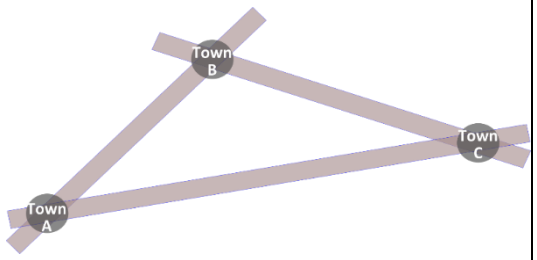


Assessing for Understanding

Students should demonstrate an understanding of the properties of the centers of triangles. The following task prompts students to consider the different centers, apply the properties to the context and make a decision about where to place the amphitheater.

Example:

A city plans to build an amphitheater and wants to locate it within easy access of the three largest towns in the area as shown on the map.



The developer must decide on the best location. The city will also have roads built for access directly to the towns or to the existing highways.

Describe how the developer might identify the location for the amphitheater. Choose one of the methods described and justify why this is the best location.

Possible student responses:
The circumcenter would place the amphitheater equidistant from the town. Roads would need to be built from the towns to the amphitheater. These roads would be the same distance.

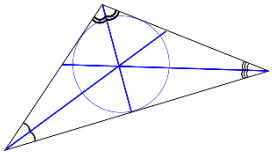
Mastering the Standard

Comprehending the Standard

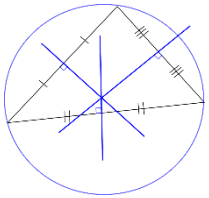
Once defined, students should experiment to verify the following properties:

- The centroid
  - always falls within the triangle
  - is located two-thirds of the way along each median or partitions the median into a ratio of 2:1 with the longest segment nearest the vertex
  - divides the triangle into six triangles of equal area
  - is the center of gravity for the triangle.

- The incenter
  - always falls within the triangle
  - equidistant from the sides of the triangle
  - is the center of the circle that is inscribed by the triangle; largest circle that will fit inside a circle and touch all three sides



- The circumcenter
  - falls inside when the triangle is acute; outside when it is obtuse, and on the hypotenuse when it is right.
  - equidistant from the vertices of the triangle
  - is the center of the circle that circumscribes the triangle; the circle that passes through all three vertices



Assessing for Understanding

The incenter would place the amphitheater from each road connecting the towns. Roads would need to be built from the existing roads to the amphitheater. These roads would be the same distance.

The centroid would place the amphitheater within the area surrounded by the three towns.

Instructional Resources

Tasks	Additional Resources
Inscribing and Circumscribing a Right Triangle NEW	

Back to: Table of Contents

NC.M3.G-CO.11

Prove geometric theorems.

Prove theorems about parallelograms.

- Opposite sides of a parallelogram are congruent.
- Opposite angles of a parallelogram are congruent.
- Diagonals of a parallelogram bisect each other.

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Concepts and Skills

Pre-requisite

- Prove theorems about lines, angles, and segments for relationships in geometric figures (NC.M2.G-CO.9)
- Use triangle congruence to prove theorems about lines, angles, and segments in triangles (NC.M2.G-CO.10)

Connections

- Apply properties, definitions, and theorems of 2-D figures to prove geometric theorems (NC.M3.G-CO.14)
- Apply geometric concepts in modeling situations (NC.M3.G.MG.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

This standard is connected to the standards NC.M2.G-CO.8 & 9. Students use the triangle congruency theorems and theorems about lines and angles to prove theorems about parallelograms. The standard includes four specific theorems; however, student experience should not be limited to only these four.

Students should prove and apply the theorems listed. Application may include using the theorems to prove other theorems or to solve problems. (connect to NC.M3.G-CO.14 and NC.M3.G-MG.1).

Given the definition of a parallelogram (a quadrilateral with both pairs of opposite sides

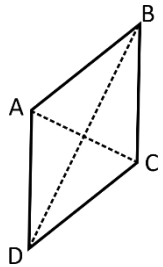
Assessing for Understanding

Students should prove theorems about parallelograms including:

- Opposite sides of a parallelogram are congruent.
- Opposite angles of a parallelogram are congruent
- Diagonals of a parallelogram bisect each other
- If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Students should apply proven theorems to prove additional theorems.

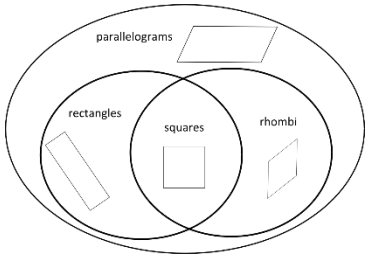
Example: Given ABCD is a rhombus prove the diagonals $\overline{BD}$ and $\overline{AC}$ are perpendicular bisectors.



Mastering the Standard

Comprehending the Standard

parallel) all other properties of a parallelogram can be proven.



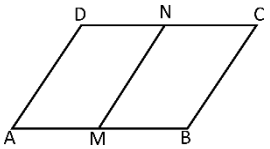
Rectangles, rhombi, and squares are specific types of parallelograms. Consider including theorems that are specific to these such as:

- Diagonals of a rhombus are perpendicular bisectors.
- Diagonals of a square are congruent and perpendicular bisectors.
- Diagonals of a rhombus bisect the vertex angles.

Proof is not solely about knowing the theorems. The goal of proof is to further develop the ability to construct logical arguments. Students should develop both *flow* and *paragraph* proofs. The construction of logical arguments and the ability to explain their reasoning is what will be expected from students.

Assessing for Understanding

Example: Suppose that ABCD is a parallelogram, and that $\overline{AB}$ and $\overline{CD}$ respectively. Prove that $\overline{MN}$ = $\overline{AD}$ and that the



M and N are the midpoints of line $\overleftrightarrow{MN}$ is parallel to $\overleftrightarrow{AD}$.

Instructional Resources

Tasks	Additional Resources
	Properties of Quadrilaterals (Inside Mathematics)

Back to: Table of Contents

NC.M3.G-CO.14

Prove geometric theorems.

Apply properties, definitions, and theorems of two-dimensional figures to prove geometric theorems and solve problems.

Concepts and Skills

Pre-requisite

- Prove theorems about parallelograms (NC.M3.G-CO.11)

Connections

- Use similarity to solve problems and to prove theorems about triangles (NC.M2.G-SRT.4)
- Understand and apply theorems about circles (NC.M3.G-C.2)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them
3 – Construct viable arguments and critique the reasoning of others
5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

This standard is the application of the other two standards within this cluster NC.M3.G-CO.10 & 11. The other standards have students determine properties and prove theorems of figures. This standard is an application of those standards.

For this standard, instruction should provide students the opportunity to prove theorems for other two dimensional figures and to reason with figures to solve problems.

The geometric theorems may be for specific defined shapes. Consider including other quadrilaterals such as trapezoids and kites for students to explore. For example, prove the base angles of an isosceles trapezoid are congruent.

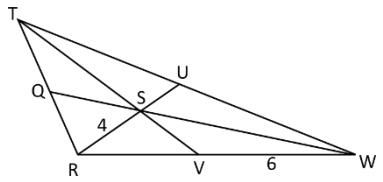
Assessing for Understanding

Students should demonstrate a solid understanding of lines and angles (Math 2), congruent triangles (Math 2), and properties of the centers of triangles (Math 3) and properties of parallelograms (Math 3). They should use their understanding of these properties, definitions and theorems to prove other geometric theorems and solve problems.

Students should use properties of the centers of triangles to solve problems.

Example: S is the centroid of ΔRTW; RS = 4, VW = 6 and TV = 9. Find the length of each segment:

a. RV b. SU c. RU
d. RW e. TS f. SV

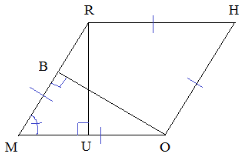


Students should use theorems about parallelograms to solve problems.

Mastering the Standard

Comprehending the Standard

The geometric theorems may also be for a specific given figure. For example, given the rhombus RHOM, prove $\overline{RU} \cong \overline{OB}$.



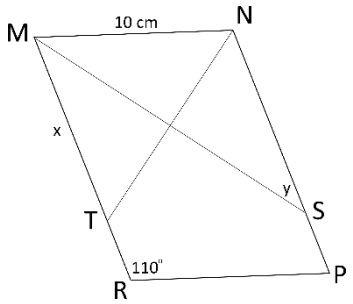
Finally, this standard should be connected to NC.M3.G-C.2 where students are understanding and applying theorems about circles.

There is not a specific list of theorems for students to know and use. The focus is not on specific theorems but on construction of logical arguments and the ability of students to explain their reasoning with two-dimensional figures.

Assessing for Understanding

Example: Given MNPR is a parallelogram, $\overline{MS}$ bisects ∠RMN and $\overline{NT}$ bisects ∠MNP

a. Find the values of x and y.
b. Describe the relationship between $\overline{MS}$ and $\overline{NT}$

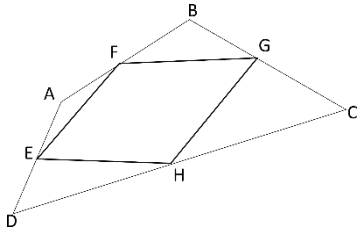


Example: In rectangle ABCD, AC = 3x + 15 and BD = 4x − 5. If AC and BD intersect at G, find the length of AG.

Students should be able to prove geometric theorems.

Example: Prove each of the following is true for an isosceles trapezoid.

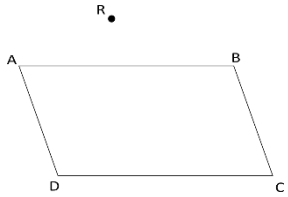
- Base angles are congruent.
- Opposite angles are supplementary.
- Diagonals are congruent.



Example: For quadrilateral ABCD, points E, F, G and H are midpoints of their respective sides. Prove EFGH is a parallelogram.

Students should be able to reason with two dimensional figures to solve problems.

Example: In figure ABCD, AB||CD and AD||BC. Point R is in the same plane as ABCD. (Point R can be placed anywhere in the plane.)



Draw a straight line that passes through point R and divides ABCD into two congruent parts. Justify your reasoning that the two parts are congruent.

Source:http://www.utdanacenter.org/k12mathbenchmarks/tasks/8_congruence.php

Instructional Resources

Tasks	Additional Resources

Back to: Table of Contents

Geometry – Circles

NC.M3.G-C.2

Understand and apply theorems about circles.

Understand and apply theorems about circles.

- Understand and apply theorems about relationships with angles and circles, including central, inscribed and circumscribed angles.
- Understand and apply theorems about relationships with line segments and circles including, radii, diameter, secants, tangents and chords.

Concepts and Skills

Pre-requisite

- Prove theorems about lines, angles, and segments for relationships in geometric figures (NC.M2.G-CO.9)
- Use similarity to solve problems and to prove theorems about triangles (NC.M2.G-SRT.4)

Connections

- Apply geometric concepts in modeling situations (NC.M3.G.MG.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them
3 – Construct viable arguments and critique the reasoning of others
5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

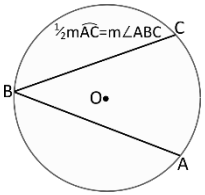
New Vocabulary: Circumscribe, inscribe, tangent

Mastering the Standard

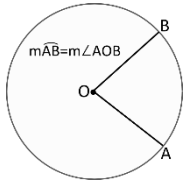
Comprehending the Standard

The following relationships with circles provide the foundation for reasoning with and applying theorems about circles:

- Relationships with angles and circles
  - **Central angle** is an angle formed by two intersecting radii such that its vertex is at the center of the circle; the measure of the angle is equal to the measure of the intersected arc



  - **Inscribed angle** is an angle with its vertex on the circle, formed by two intersecting chords; the measure of the angle is half the measure of the intersected arc



Assessing for Understanding

Students should have a strong command of the vocabulary: central angle, inscribed angle, circumscribed angle, tangent, arc (minor & major), secant, and chord.

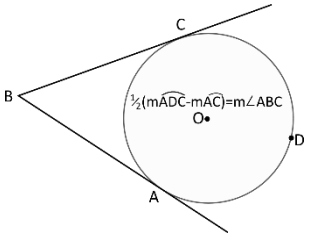
Students demonstrate understanding when applying theorems about circles to explore other theorems.

- an angle inscribed in a semi-circle is a right angle.
- the opposite angles in an inscribed quadrilateral are supplementary.
- tangent lines drawn from a point outside a circle are equal in length.
- when two chords intersect at a point interior to a circle, the chords are divided proportionally.
- when two secants intersect at a point exterior to a circle, the lengths of the secants and the external parts are proportional.
- if two chords are equivalent then their minor arcs are congruent and conversely
- if two chords are equidistant from the center then they are congruent and conversely

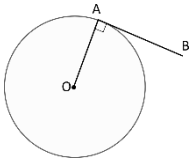
Mastering the Standard

Comprehending the Standard

- **Circumscribed angle** is an angle formed by two tangents to a circle from the same point outside the circle; the measure of the angle is half the difference of the intercepted arcs



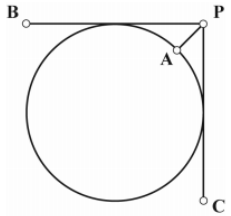
- Relationships with line segments and circles:
  - **Tangent line** intersects the circle exactly once at the point of tangency; the tangent line is perpendicular to the radius at the point of tangency



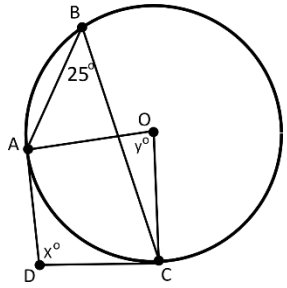
Assessing for Understanding

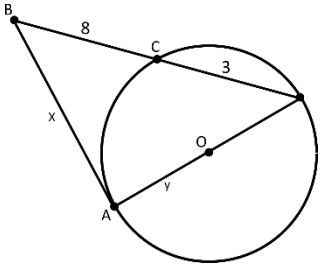
Students demonstrate understanding when applying theorems about circles to solve problems with and without context.

Example: A round table is pushed into a corner. The diameter of the table is 5 feet. Find the distance from the corner to the edge of the table



Example: Find the value of x and y.





Instructional Resources

Tasks	Additional Resources

Back to: Table of Contents

Geometry – Circles

NC.M3.G-C.5

Understand and apply theorems about circles.

Using similarity, demonstrate that the length of an arc, *s*, for a given central angle is proportional to the radius, *r*, of the circle. Define radian measure of the central angle as the ratio of the length of the arc to the radius of the circle, *s/r*. Find arc lengths and areas of sectors of circles.

Concepts and Skills

Pre-requisite

- Know the formulas for the area and circumference of a circle and use them to solve problems (7.G.4)
- Verify the properties of dilations with given center and scale factor (NC.M2.G-SRT.1)

Connections

- Understand radian measure as domain for trigonometric functions (NC.M3.G-TF.1)
- Apply geometric concepts in modeling situations (NC.M3.G-MG.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

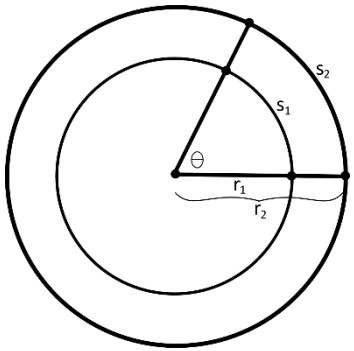
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

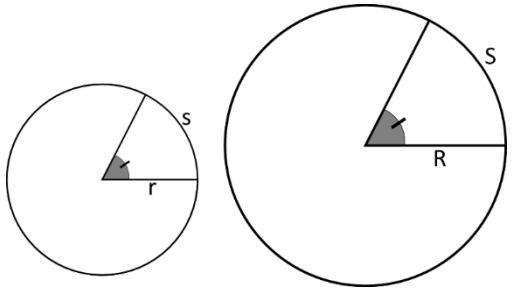
Circles are similar figures; thus, any two arcs, subtended by the same central angle, will be proportional.

Since corresponding parts of similar figures are proportional then $\frac{r_1}{r_2} = \frac{s_1}{s_2}$ which can also be be written as $s_1 = \left(\frac{s_2}{r_2}\right) r_1$. The structure of the equation reveals that the length of the arc is directly proportional to the radius and $\frac{s_2}{r_2}$ is the constant of proportionality.



Assessing for Understanding

Students demonstrate an understanding of the proportional relationship between the length of an arc and the radius of the circle by explaining how the following two diagrams could be used to prove that $s = kr$ where $k = \frac{S}{R}$ which is the radian measure of the central angle.



Students should use the definition of a radian to answer and solve problems.

Example: Explain why there are 2π radians in a circle. *Students explain that the radian measure is the ratio of the total length of the circle, 2πr, to the radius r. Thus* $\frac{2\pi r}{r} = 2\pi$ *radians.*

Example: The length of an arc is 18 cm and the radius of the circle is 6cm. What is the radian measure of the central angle?

Mastering the Standard

Comprehending the Standard

Furthermore, a radian is defined as the ratio of the length of the arc to the radius of the circle, $\frac{s}{r}$, so the constant of proportionality is the radian measure of the angle.

Using the reasoning presented, the arc length, *s*, can be calculated using the formula $s = \theta r$ where θ is the radian measure and *r* is the radius of the circle.

The length of an arc subtended by a central angle can also be expressed as a fraction of the circumference. Given the central angle θ in degrees, the arc length is $s = \frac{\theta}{360^\circ}(2\pi r)$. Given the central angle θ in radians, the arc length is $s = \frac{\theta}{2\pi}(2\pi r) = \theta r$.

Similarly, the area of a sector can be expressed as a fraction of the area of the circle. Given the central angle in degrees and the radius *r*, the area of a sector is $\frac{\theta}{360^\circ}(\pi r^2)$. Given the central angle in radians and the radius *r*, the area of the sector is $\frac{\theta}{2\pi}(\pi r^2) = \frac{\theta}{2}r^2 = \frac{sr}{2}$ where *s* is the arc length.

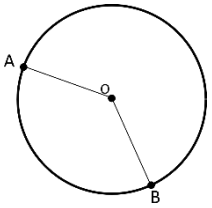
Assessing for Understanding

Example: A central angle measures 4.5 radians and has an arc length of 35 inches. What is the radius of the circle?

Students should be able to calculate arc lengths and areas of sectors of circles.

Example: Given that $m\angle AOB = \frac{2\pi}{3} radians$ and the radius is 18 cm, what is the length of AB ?

Example: Find the area of a sector with an arc length of 40 cm and a radius of 12 cm.



Instructional Resources

Tasks	Additional Resources

Back to: Table of Contents

Geometry – Expressing Geometric Properties with Equations

NC.M3.G-GPE.1

Translate between the geometric description and the equation for a conic section.

Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Concepts and Skills

Pre-requisite

- Apply the Pythagorean Theorem to find the distance between two points (8.G.8)
- Write an equivalent form of a quadratic expression by completing the square (NC.M2.A-SSE.3)

Connections

- Work with conic sections (4th level course)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

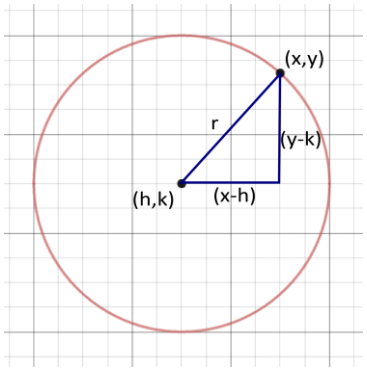
Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

Students derive the standard equation of a circle by reasoning with circles on the coordinate plane. Given a center (h, k) and a radius r, students determine that the horizontal distance from the center to a point (x, y) on the circle can be expressed by $(x - h)$. Likewise, the vertical distance from the center to the point can be expressed by $(y - k)$. These distances can be modeled by a vertical and horizontal line segment. The radius can be modeled by a line segment connecting the center to the point. A right triangle is formed and the Pythagorean Theorem can be applied to derive $(x - h)^2 + (y - k)^2 = r^2$.



For a circle equation in general form $x^2 + y^2 + cx + dx + e = 0$, students will use the process of completing the square to rewrite and identify the center and radius of the circle. (The process of completing the square is in Math 2 NC.M2.A-SSE.3.)

Assessing for Understanding

Students demonstrate an understanding of the equation of a circle by writing the equation using the center and radius.

Example: Write the equation of a circle that is centered at (-1,3) with a radius of 5 units.

Example: Using the whole numbers 1 – 9 as many times as you like, make the biggest circle by filling in the blanks below:

$$\square x^2 + \square y^2 = \square$$

Source: http://www.openmiddle.com/make-the-biggest-circle/

Example: Write an equation for a circle given that the endpoints of the diameter are (-2,7) and (4,-8)

Example: How many points with two integer coordinates are 5 units away from (-2, 3)?

Source: http://www.openmiddle.com/equidistant-points/

Students can rewrite the equation of a circle to identify the center and radius.

Example: Find the center and radius of the circle $4x^2 + 4y^2 - 4x + 2y - 1 = 0$.

Instructional Resources	
Tasks	**Additional Resources**
Explaining the Equation of a Circle (Illustrative Mathematics) Sorting the Equations of a Circle 1 (MathShell) NEW Sorting the Equations of a Circle 2 (MathShell) NEW	

Back to: Table of Contents

NC.M3.G-GMD.3

Explain volume formulas and use them to solve problems.

Use the volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems.

Concepts and Skills

Pre-requisite

- Know and use formulas for volumes of cones, cylinders, and spheres (8.G.9)

Connections

- Solve for a quantity of interest in formulas (NC.M1.A-CED.4)
- Apply geometric concepts in modeling situations (NC.M3.G-MG.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

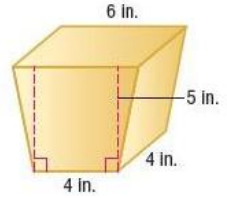
This standard focuses on volume and the use of volume formulas to solve problems. The figures may be a single shape or a composite of shapes.

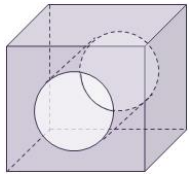
Formulas should be provided as the figures are more complex and the focus is on the modeling and solving problems.

Assessing for Understanding

Students should be able to identify the 3-D figures (prisms, cylinders, pyramids, cones and spheres) and the measurements needed to calculate the volume.

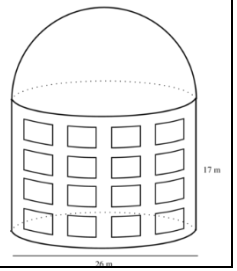
Example: A carryout container is shown. The bottom base is a 4-inch square and the top base is a 4-inch by 6-inch rectangle. The height of the container is 5 inches. Find the volume of food that it holds.





Example: A toy manufacture has designed a new piece for use in building models. It is a cube with side length 7 mm and it has a 3 mm diameter circular hole cut through the middle. The manufacture wants 1,000,000 prototypes. If the plastic used to create the piece costs $270 per cubic meter, how much will the prototypes cost?

Example: The Southern African Large Telescope (SALT) is housed in a cylindrical building with a domed roof in the shape of a hemisphere. The height of the building wall is 17 m and the diameter is 26 m. To program the ventilation system for heat, air conditioning, and dehumidifying, the engineers need the amount of air in the building. What is the volume of air in the building?



Instructional Resources

Tasks	Additional Resources
Cylinders (OpenMiddle.com)	
Calculating Volumes of Compound Objects NEW	

Geometry – Geometric Measurement & Dimension

NC.M3.G-GMD.4

Visualize relationships between two-dimensional and three-dimensional objects.

Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Concepts and Skills

Pre-requisite

- Describe 2-D cross-sections of rectangular prisms and pyramids (7.G.3)

Connections

- Apply geometric concepts in modeling situations (NC.M3.G-MG.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

This standard has two parts.

The first part is to identify the two-dimensional cross sections of three-dimensional objects.

Consider having students work with manipulatives such as play-dough and floss to make slices of three-dimensional shapes. Also, the *Cross Section Flyer* at http://www.shodor.org/interactivate/activities/CrossSectionFlyer/ can be used to allow students to predict and verify the cross section of different three-dimensional objects.

The second part is identifying three-dimensional objects generated by rotations of two-dimensional objects. There are a few interactive websites that students can use to explore.

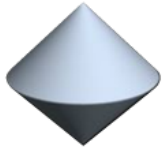
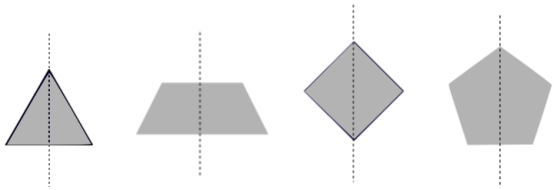
- 3D Transmographer http://www.shodor.org/interactivate/activities/3DTransmographer/
- Interactive Tool: Stacker http://www.scootle.edu.au/ec/viewing/L588/index.html
- Interactive Tool: Replicator http://www.scootle.edu.au/ec/viewing/L1059/index.html

Assessing for Understanding

Students identify shapes of two-dimensional cross-sections of three-dimensional objects.

Example: Draw a figure that has the same cross section as a sphere.

Example: Which of the following is the cross section created by slicing the cylinder as shown in the figure?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding** Students identify three-dimensional objects generated by rotations of two-dimensional objects. **Example:** The shape at the right was created by rotating a two dimensional shape about an axis. Which of the following would create this shape?  

Instructional Resources	
Tasks	**Additional Resources**
Representing 3D objects in 2D NEW	

Back to: Table of Contents

Geometry – Modeling with Geometry

NC.M3.G-MG.1

Apply geometric concepts in modeling situations.

Apply geometric concepts in modeling situations

- Use geometric and algebraic concepts to solve problems in modeling situations:
- Use geometric shapes, their measures, and their properties, to model real-life objects.
- Use geometric formulas and algebraic functions to model relationships.
- Apply concepts of density based on area and volume.
- Apply geometric concepts to solve design and optimization problems.

Concepts and Skills

Pre-requisite

- Solve real world problems involving area, volume, and surface area (7.G.6)
- Use volume formulas to solve problems (NC.M3.G-GMD.3)

Connections

- Apply properties, definitions, and theorems of 2-D figures to solve problems (NC.M3.G-CO.14)
- Understand and apply theorems about circles (NC.M3.G-C.2)
- Find arc lengths and areas of sectors of circles (NC.M3.G-C.5)
- Identify 2-D cross sections; identify 3-D objects (NC.M3.G-GMD.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

For this standard, students should engage in problems that are more complex than those studied in previous grades. The standard combines geometric and algebraic concepts and focuses on four primary areas:

i. model real-world three-dimensional figures,
ii. model relationships,
iii. determine density based on area or volume, and
iv. solve design and optimization problems.

When students model real-world three dimensional figures they must recognize the plane shapes that comprise the figure. They must be flexible in constructing and deconstructing the shapes. Students also need to be able to identity the measures associated with the figure such as circumference, area, perimeter, and volume.

Assessing for Understanding

Students recognize situations that require relating two- and three- dimensional objects. They estimate measures (circumference, area, perimeter, volume) of real-world objects using comparable geometric shapes or three-dimensional objects. Students apply the properties of geometric figures to comparable real-world objects (e.g., The spokes of a wheel of a bicycle are equal lengths because they represent the radii of a circle).

Use geometric and algebraic concepts to solve problems in modeling situations.

Example: Janine is planning on creating a water-based centerpiece for each of the 30 tables at her wedding reception. She has already purchased a cylindrical vase for each table.

- The radius of the vases is 6 cm and the height is 28 cm.
- She intends to fill them half way with water and then add a variety of colored marbles until the waterline is approximately three-quarters of the way up the cylinder.
- She can buy bags of 100 marbles in 2 different sizes, with radii of 9mm or 12 mm. A bag of 9 mm marbles costs $3, and a bag of 12 mm marbles costs $4.

Mastering the Standard

Comprehending the Standard

Students use formulas and algebraic functions when modeling relationships. This may include examining how the one measurement changes as another changes.

How does the volume of a cylinder change as the radius changes?

How does the surface area of a prism change as the height changes?

The concept of density based on area and volume is to calculate the mass per unit.

Examples for area density are:

Description	*Unit of Measure*
Data Storage	Gigabytes per square inch
Thickness of Paper	Grams per square meter
Bone density	Grams per square centimeter
Body Mass Index	Kilograms per square meter
Population	People per square mile

Examples for volume density are:

Description	*Unit of Measure*
Solids	Grams per cubic centimeter
Liquids	Grams per millliter (1 mL = 1 cubic cm)

Design problems include designing an object to satisfy physical constraints. Optimization problems may maximize or minimize depending on the context.

Assessing for Understanding

a. If Janine only bought 9 mm marbles how much would she spend on marbles for the whole reception? What if Janine only bought 12 mm marbles? (Note: 1 cm^3 = 1 mL)

b. Janine wants to spend at most d dollars on marbles. Write a system of equalities and/or inequalities that she can use to determine how many marbles of each type she can buy.

c. Based on your answer to part b. How many bags of each size marble should Janine buy if she has $180 and wants to buy as many small marbles as possible?

Geometric shapes, their measures, and their properties to model real-life objects

Example: Describe each of the following as a simple geometric shape or combination of shapes. Illustrate with a sketch and label dimensions important to describing the shape.

a. Soup can label
b. A bale of hay
c. Paperclip
d. Strawberry

Use geometric formulas and algebraic functions to model relationships.

Example: A grain silo has the shape of a right circular cylinder topped by a hemisphere. If the silo is to have a capacity of 614π cubic feet, find the radius and height of the silo that requires the least amount of material to construct.

Density based problems

Example: A King Size waterbed has the following dimensions 72 in. x 84 in. x 9.5in. It takes 240.7 gallons of water to fill it, which would weigh 2071 pounds. What is the weight of a cubic foot of water?

Example: Wichita, Kansas has 344,234 people within 165.9 square miles. What is Wichita's population density?

Instructional Resources

Tasks	Additional Resources

Back to: Table of Contents

Statistics & Probability

A statistical process is a problem-solving process consisting of four steps:

1. Formulating a statistical question that anticipates variability and can be answered by data.
2. Designing and implementing a plan that collects appropriate data.
3. Analyzing the data by graphical and/or numerical methods.
4. Interpreting the analysis in the context of the original question.

NC Math 1	NC Math 2	NC Math 3
Focus on analysis of univariate and bivariate data • Use of technology to represent, analyze and interpret data • Shape, center and spread of univariate numerical data • Scatter plots of bivariate data • Linear and exponential regression • Interpreting linear models in context.	**Focus on probability** • Categorical data and two-way tables • Understanding and application of the Addition and Multiplication Rules of Probability • Conditional Probabilities • Independent Events • Experimental vs. theoretical probability	**Focus on the use of sample data to represent a population** • Random sampling • Simulation as it relates to sampling and randomization • Sample statistics • Introduction to inference
A Progression of Learning		
• A continuation of the work from middle grades mathematics on summarizing and describing quantitative data distributions of univariate (6^{th} grade) and bivariate (8^{th} grade) data.	• A continuation of the work from 7^{th} grade where students are introduced to the concept of probability models, chance processes and sample space; and 8^{th} grade where students create and interpret relative frequency tables. • The work of MS probability is extended to develop understanding of conditional probability, independence and rules of probability to determine probabilities of compound events.	• Bringing it all back together • Sampling and variability • Collecting unbiased samples • Decision making based on analysis of data

Back to: Table of Contents

NC.M3.S-IC.1

Understand and evaluate random processes underlying statistical experiments.

Understand the process of making inferences about a population based on a random sample from that population.

Concepts and Skills

Pre-requisite

- Use data from a random sample to draw inferences about a population (7.SP.2)

Connections

- Recognize the purpose and differences between samples and studies and how randomization is used (NC.M3.S-IC.3)
- Use simulation estimate a population mean or proportion (NC.M3.S-IC.4)
- Use simulation to determine whether observed differences between samples indicate the two populations are distinct (NC.M3.S-IC.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: sample, population, random sample, inferential statistics

Mastering the Standard

Comprehending the Standard

The statistical process includes four essential steps:

1. Formulate a question that can be answered with data.
2. Design and use a plan to collect data.
3. Analyze the data with appropriate methods.
4. Interpret results and draw valid conclusions.

An essential understanding about the data collection step is that random selection can produce samples that represent the overall population. This allows for the generalization from the sample to the larger population in the last step of the process.

A *population* consists of everything or everyone being studied in an inference procedure. It is rare to be able to perform a census of every individual member of the population. Due to constraints of resources it is nearly impossible to perform a measurement on every subject in a population.

Assessing for Understanding

Students demonstrate an understanding of the different kinds of sampling methods.

Example: From a class containing 12 girls and 10 boys, three students are to be selected to serve on a school advisory panel. Here are four different methods of making the selection.

a. Select the first three names on the class roll.
b. Select the first three students who volunteer.
c. Place the names of the 22 students in a hat, mix them thoroughly, and select three names from the mix.
d. Select the first three students who show up for class tomorrow.

Which is the best sampling method, among these four, if you want the school panel to represent a fair and representative view of the opinions of your class? Explain the weaknesses of the three you did not select as the best.

Students should recognize the need for random selection, describe a method for selecting a random sample from a given population, and explain why random assignment to treatments is important in the design of a statistical experiment.

Example: A department store manager wants to know which of two advertisements is more effective in increasing sales among people who have a credit card with the store. A sample of 100 people will be selected from the 5,300 people who have a credit card with the store. Each person in the sample will be called and read one of the two advertisements. It will then be determined if the credit card holder makes a purchase at the department store within two weeks of receiving the call.

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
A *random sample* is a sample composed of selecting from the population using a chance mechanism. Often referred to as a simple random sample. *Inferential statistics* considers a subset of the population. This subset is called a statistical sample often including members of a population selected in a random process. The measurements of the individuals in the sample tell us about corresponding measurements in the population.	a. Describe the method you would use to determine which credit card holders should be included in the sample. Provide enough detail so that someone else would be able to carry out your method. b. For each person in the sample, the department store manager will flip a coin. If it lands heads up, advertisement A will be read. If it lands tails up, advertisement B will be read. Why would the manager use this method to decide which advertisement is read to each person? Source: https://locus.statisticseducation.org/

Instructional Resources	
Tasks	**Additional Resources**
Performance Task (LearnZillion.com)	

Back to: Table of Contents

NC.M3.S-IC.3

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Recognize the purposes of and differences between sample surveys, experiments, and observational studies and understand how randomization should be used in each.

Concepts and Skills

Pre-requisite

- Understand the process of making inferences (NC.M3.S-IC.1)

Connections

- Use simulation estimate a population mean or proportion (NC.M3.S-IC.4)
- Use simulation to determine whether observed differences between samples indicate the two populations are distinct (NC.M3.S-IC.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: Observational study, simulation, sample, population, random sample, inferential statistics

Mastering the Standard

Comprehending the Standard

Students understand the different methods of data collection, specifically the difference between an observational study and a controlled experiment, and know the appropriate use for each.

- *Observational study* – a researcher collects information about a population by measuring a variable of interest, but does not impose a treatment on the subjects. (i.e. examining the health effects of smoking)

- *Experiment* – an investigator imposes a change or treatments on one or more group(s), often called treatment group(s). A comparative experiment is where a control group is given a placebo to compare the reaction(s) between the treatment group(s) and the control group.

Assessing for Understanding

Students should be able to distinguish between the different methods.

A student wants to determine the most liked professor at her college. Which type of study would be the most practical to obtain this information?

a) simulation
b) experiment
c) survey
d) observation

Source: NC Measure of Student Learning CC Math III Spring 2013

Students understand the role that randomization plays in eliminating bias from collected data.

Example: Students in a high school mathematics class decided that their term project would be a study of the strictness of the parents or guardians of students in the school. Their goal was to estimate the proportion of students in the school who thought of their parents or guardians as "strict". They do not have time to interview all 1000 students in the school, so they plan to obtain data from a sample of students.

a) Describe the parameter of interest and a statistic the students could use to estimate the parameter.
b) Is the best design for this study a sample survey, an experiment, or an observational study? Explain your reasoning.

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
	c) The students quickly realized that, as there is no definition of "strict", they could not simply ask a student, "Are your parents or guardians strict?" Write three questions that could provide objective data related to strictness. d) Describe an appropriate method for obtaining a sample of 100 students, based on your answer in part (a) above.

Instructional Resources	
Tasks	**Additional Resources**

Back to: Table of Contents

NC.M3.S-IC.4

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Use simulation to understand how samples can be used to estimate a population mean or proportion and how to determine a margin of error for the estimate.

Concepts and Skills

Pre-requisite

- Design and use simulation to generate frequencies for compound events (7.SP.8c)
- Understand the process of making inferences (NC.M3.S-IC.1)

Connections

- Recognize the purpose and differences between samples and studies and how randomization is used (NC.M3.S-IC.3)
- Use simulation to determine whether observed differences between samples indicate the two populations are distinct (NC.M3.S-IC.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: simulation, sample, population, margin of error, parameter

Mastering the Standard

Comprehending the Standard

This standard has two parts:

1. Use simulation to understand how samples can be used to estimate a population mean or proportion
2. Use simulation to determine a margin of error for the estimate

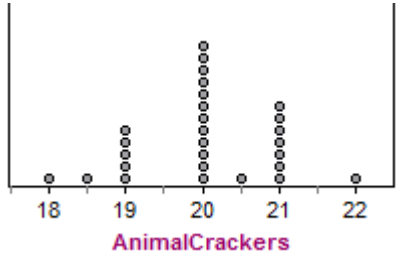
Simulations may use physical manipulatives: dice, cards, beads, decks of playing cards. If available, simulations can be completed using technology. In either situation, students should have a clear understanding of how the simulation models the situation. For the first part, students understand that a sample only provides an estimate of the population parameter. With repeated sampling, the estimates vary and a sampling distribution can be created to model the variation.

Consider trying to determine the proportion of orange candies in Reese's Pieces. After taking a sample of 25 pieces, the proportion of orange is 0.40. Another sample has a proportion of orange as 0.60. By taking 100 random samples and computing the proportion of orange for each one a sampling distribution can be made.

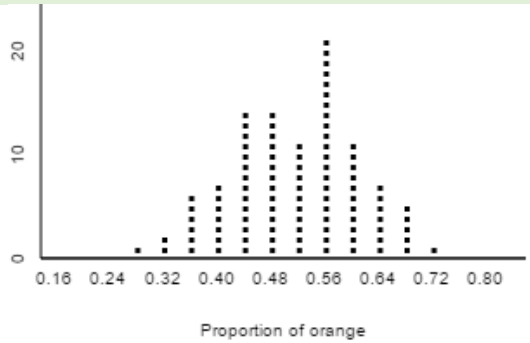
Assessing for Understanding

Students should use a simulation to estimate a population mean or proportion and determine a margin of error for that estimate.

Example: The label on a Barnum's Animal Cracker box claims that there are 2 servings per box and a serving size is 8 crackers. The graph displays the number of animal crackers found in a sample of 28 boxes. Use the data from the 28 samples to estimate the average number of crackers in a box with a margin of error. Explain your reasoning or show your work.



Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**



Using the sampling distribution, students can estimate a population proportion using the mean of the distribution (0.51).

Simulation for Reese's Pieces at
http://www.rossmanchance.com/applets/OneProp/OneProp.htm?candy=1

For the second part, students understand that the margin of error is the maximum likely mistake in prediction. In other words, it is the most that a value of a sample statistic is likely to differ from the actual value of the population parameter.
One informal way of developing a margin of error from a simulation is to compute the value that is the range of the simulation's results divided by 2 ($margin\ of\ error = \frac{range}{2}$).

In the Reese's Pieces simulation, the margin of error would be 0.22 meaning the actual population proportion of orange candies is between 0.29 and 0.73.

Taking larger sample sizes would decrease the margin of error.

Changing the sample size to 50 gives a margin of error of 0.15.

Margin of error can be computed by formula; however, this standard is intended to engage students in using simulations to estimate. Note that confidence intervals are beyond what is intended in the standard. Students should have an idea of what margin of error is and how it is interpreted, which can lead informally to the idea of an interval estimate.

Instructional Resources	
Tasks	**Additional Resources**
Scratch 'N Win Blues (Illustrative Mathematics) Margin of Error for Estimating a Population Mean (Illustrative Mathematics)	

Back to: Table of Contents

NC.M3.S-IC.5

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Use simulation to determine whether observed differences between samples from two distinct populations indicate that the two populations are actually different in terms of a parameter of interest.

Concepts and Skills

Pre-requisite

- Design and use simulation to generate frequencies for compound events (7.SP.8c)
- Understand the process of making inferences (NC.M3.S-IC.1)

Connections

- Recognize the purpose and differences between samples and studies and how randomization is used (NC.M3.S-IC.3)
- Use simulation estimate a population mean or proportion (NC.M3.S-IC.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: simulation, sample, population, parameter

Mastering the Standard

Comprehending the Standard

The statistical process includes four essential steps:

1. Formulate a question that can be answered with data.
2. Design and use a plan to collect data.
3. Analyze the data with appropriate methods.
4. Interpret results and draw valid conclusions.

This standard addresses parts 3 and 4 of this process. Once data is collected from an experiment, it is necessary to determine if there are differences between the two treatment groups. If so, are the differences due to the treatment or due to variation within the population?

Select a sample statistic to compare. For example, the mean of each sample.

Consider the experiment where twenty male students were randomly assigned to one of two treatment groups of 10 students each, one group receiving 200 milligrams of caffeine and the other group no caffeine.

The parameter of interest is the number of finger taps per minute. The sample statistics showed that the mean of the 200

Assessing for Understanding

Students should demonstrate an understanding of the process by

- identifying the parameter of interest,
- select and calculate sample statistics,
- calculate the difference between the sample statistic,
- set up and complete a simulation re-randomizing the groups,
- and compare the actual difference to the simulated differences

Example: Sal purchased two types of plant fertilizer and conducted an experiment to see which fertilizer would be best to use in his greenhouse. He planted 20 seedlings and used Fertilizer A on ten of them and Fertilizer B on the other ten. He measured the height of each plant after two weeks. Use the data below to determine which fertilizer Sal should use.

Fertlizer A	23.4	30.1	28.5	26.3	32.0	29.6	26.8	25.2	27.5	30.8
Fertlizer B	19.8	25.7	29.0	23.2	27.8	31.1	26.5	24.7	21.3	25.6

a. Use the data to generate simulated treatment results by randomly selecting ten plant heights from the twenty plant heights listed.

b. Calculate the average plant height for each treatment of ten plants.

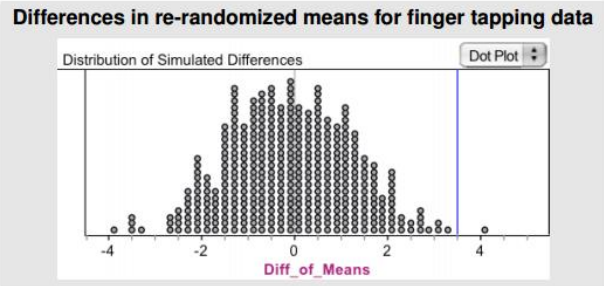
Mastering the Standard

Comprehending the Standard

mg group was 3.5 taps more than the 0 mg group. Thus, an observed difference.

Use simulation to determine if the observed difference is due to the caffeine.

Is it possible that the 3.5 taps was due to randomization and not caffeine? In order to find out, re-randomize the participants and calculate the difference in means. Simulate this and create a distribution of the results.



The results of the simulation shows that the difference of 3.5 is equaled or exceeded only once out of 400 trials this providing strong evidence that the caffeine is the cause of the increased tapping.

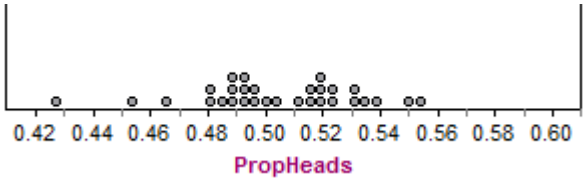
Source: http://commoncoretools.me/wp-content/uploads/2012/06/ccss_progression_sp_hs_2012_04_21_bis.pdf

Assessing for Understanding

c. Find the difference between consecutive pairs of treatment averages and compare. Does your simulated data provide evidence that the average plant heights using Fertilizer A and Fertilizer B is significant?

Example: "Are Starbucks customers more likely to be female?" To answer the question, students decide to randomly select 30-minute increments of time throughout the week and have an observer record the gender of every tenth customer who enters the Starbucks store. At the end of the week, they had collected data on 260 customers, 154 females and 106 males. This data seems to suggest more females visited Starbucks during this time than males.

To determine if these results are statistically significant, students investigated if they could get this proportion of females just by chance if the population of customers is truly 50% females and 50% males. Students simulated samples of 260 customers that are 50-50 females to males by flipping a coin 260 then recording the proportion of heads to represent the number of women in a random sample of 260 customers (e.g., 0.50 means that 130 of the 260 flips were heads). Their results are displayed in the graph at the right.



Use the distribution to determine if the class's data is statistically significant enough to conclude that Starbucks customers are more likely to be female.

Instructional Resources

Tasks	Additional Resources

Back to: Table of Contents

Statistics & Probability – Making Inference and Justifying Conclusions

NC.M3.S-IC.6

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Evaluate articles and websites that report data by identifying the source of the data, the design of the study, and the way the data are graphically displayed.

Concepts and Skills

Pre-requisite

- Use appropriate statistics to compare center and spread of two or more data sets and interpret differences in context (NC.M1.S-ID.2)
- Recognize the purpose and differences between samples and studies and how randomization is used (NC.M3.S-IC.3)

Connections

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

The statistical process includes four essential steps:

1. Formulate a question that can be answered with data.
2. Design and use a plan to collect data.
3. Analyze the data with appropriate methods.
4. Interpret results and draw valid conclusions.

When students are presented with information supported by data, they should critically examine the source of the data, the design of the study and the graphs to determine the validity of the article or website.

Students should recognize how graphs and data can be distorted to support different points of view. Students should use spreadsheet tables and graphs or graphing technology to recognize and analyze distortions in data displays.

This standard connects to NC.M3.S-IC.1, 3, 4, & 5.

Assessing for Understanding

Students critically evaluate the source of the data, the design of the study, and the graphical displays.

Example: Read the article below from NPR.org then answer the following questions.

Kids and Screen Time: What Does the Research Say?
By Juana Summers
August 28, 2014

Kids are spending more time than ever in front of screens, and it may be inhibiting their ability to recognize emotions, according to new research out of the University of California, Los Angeles.

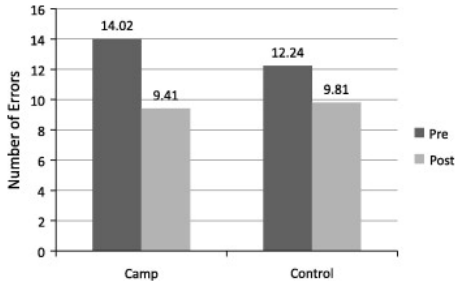
The study, published in the journal Computers in Human Behavior, found that sixth-graders who went five days without exposure to technology were significantly better at reading human emotions than kids who had regular access to phones, televisions and computers.

The UCLA researchers studied two groups of sixth-graders from a Southern California public school. One group was sent to the Pali Institute, an outdoor education camp in Running Springs, Calif., where the kids had no access to electronic devices. For the other group, it was life as usual.

At the beginning and end of the five-day study period, both groups of kids were shown images of nearly 50 faces and asked to identify the feelings being modeled. Researchers found that the students who went to camp scored significantly

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**

higher when it came to reading facial emotions or other nonverbal cues than the students who continued to have access to their media devices.



"We were pleased to get an effect after five days," says Patricia Greenfield, a senior author of the study and a distinguished professor of psychology at UCLA. "We found that the kids who had been to camp without any screens but with lots of those opportunities and necessities for interacting with other people in person improved significantly more."

If the study were to be expanded, Greenfield says, she'd like to test the students at camp a third time — when they've been back at home with smartphones and tablets in their hands for five days.

"It might mean they would lose those skills if they weren't maintaining continual face-to-face interaction," she says.

a. What is the source of the data?
b. Describe the design of the study.

After analyzing the graph, evaluate the claim that the "kids who had been to camp … improved significantly more."

Instructional Resources	
Tasks	**Additional Resources**

Back to: Table of Contents